مهتاب قدیری

۹

۸ $\log_n^{m^2 n} = \log_n^{m^2} + \log_n^n = 2\log_n^m + 1$ — ۴۱

۹ $b = 2a+1 \Rightarrow [b] = [1+2a]$ (۰٫۵)

۱۰ ۴۲

الف) $x > 0 \;/\; x \neq 1 \quad (\log_{\frac{1}{2}}^1 = 0)$

$\frac{x}{\log_{\frac{1}{2}}^x} \geq 0 \xrightarrow{x>0} \log_{\frac{1}{2}}^x > 0 \Rightarrow \boxed{0 < x < 1}$

۱۱ $\log_{\frac{1}{2}}^x < 0 \Rightarrow x > 1$ (۰٫۵)

۱۲ $D = (0,1)$

ب) $\sqrt{x^{2-1}+1} = \sqrt{x^2} = |x|$

۱۳ $x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow$ (+ | − | +) at $-1, 2$

$|x| \neq 0 \Rightarrow x \neq 0 \Rightarrow D = (-\infty,-1) \cup (2,+\infty)$

۱۴ ۴۳

۱۵ $3\log_a^x + \log_a^{\sqrt{x}} = 2 \;,\; x = 9$

$\log_a^{\sqrt{x}} = \frac{1}{2}\log_a^x \Rightarrow 3\log_a^x + \frac{1}{2}\log_a^x = 2 \Rightarrow \frac{7}{2}\log_a^x = 2$

$\frac{7}{2}\log_a^9 = 2 \Rightarrow \log_a^9 = \frac{4}{7} \Rightarrow a^{\frac{4}{7}} = 9 \overset{(\,)^{\frac{7}{4}}}{\Rightarrow} a = 9^{\frac{7}{4}} = 3^{\frac{14}{4}} = 3^{\frac{7}{2}}$ (۰٫۵)

$a = 3^{\frac{7}{2}} = \boxed{27\sqrt{3}}$

۱۷ ۴۴

$\log\frac{a}{2} = \log a - \log 2 \;,\; \log a = \log\frac{10}{2} = 1 - \log 2 = 1 - 0{,}3 = 0{,}7$

۱۸ $\log\frac{a}{2} = 0{,}7 - 0{,}3 = 0{,}4$

$\log 9 = \log 3^2 = 2\log 3 = 2(0{,}4) = 0{,}8$

$\log 15 = \log 3\times 5 = \log 3 + \log 5 = 0{,}4 + 0{,}7 = 1{,}1$

$\Rightarrow 0{,}4x^2 + 0{,}8x - 1{,}1 = 0 \overset{\times 10}{\Rightarrow} 4x^2 + 8x - 11 = 0$

$|x_1 - x_2| = \frac{\sqrt{b^2 - 4ac}}{|a|} = \frac{\sqrt{64 + 176}}{4} = \sqrt{15}$ (۱)

۵- $\log_2^5 = \log_2^{\frac{10}{2}} = \log_2^{10} - \log_2^2$ (۱)

$\hookrightarrow \log_5^2 = \frac{1}{0/5} = 2$

$\log_2^{14} = \log_2^{2\times 7} = 1 + \log_2^7 = 1 + 2/8 = 3/8$

$\log_5^{14} = \frac{2}{3/8} = \frac{10}{19} \simeq 0/53$

۶- $\log_4^5 = \frac{\log^5}{\log 4} = \frac{1}{5} \;/\; \log 4 = 2\log 2 \Rightarrow \frac{\log^5}{2\log 2} = \frac{1}{5} \Rightarrow \log^5 = \frac{2}{5}\log 2$

$\log_2^3 = \frac{\log^3}{\log 2} = \frac{1}{4} \Rightarrow \log 3 = \frac{1}{4}\log 2$

$\log 4 = \log^2 + \log^3 = \log^2 + \frac{1}{4}\log^2 = \frac{5}{4}\log^2$

$\log 75 = \log^3 \times 2\log^5 = \frac{1}{4}\log 2 + \frac{2}{5}\log 2 = \frac{29}{20}\log 2$ (۰/۵)

$\log_{75}^4 = \frac{\log 4}{\log 75} = \frac{\frac{5}{4}\log 2}{\frac{29}{20}\log 2} = \frac{5}{4} \times \frac{20}{29} = \frac{25}{29} \simeq 1/21$

۷- $0/4 = \frac{2}{5} \;/\; \frac{125}{8} = \frac{5^3}{2^3} = (\frac{5}{2})^3 \Rightarrow (\frac{2}{5})^{2x-1} = (\frac{5}{2})^{3x^2}$

$\frac{2}{5} = (\frac{5}{2})^{-1} \Rightarrow (\frac{5}{2})^{-(2x-1)} = (\frac{5}{2})^{3x^2} \Rightarrow -2x+1 = 3x^2 \Rightarrow x = \frac{1}{3}$ یا $x = -1$ (۵)

$x = \frac{1}{3} \Rightarrow 9(\frac{1}{3}) + 1 = 3 + 1 = 4$ ✓ $\quad x = -1 \Rightarrow 9(-1) + 1 = -8$ ✗ داخل لگ منفی

$8 = 2^3,\ 4 = 2^2 \Rightarrow \log_8^4 = \frac{2}{3}$

۹- $\log_8^b = \frac{\log_2^b}{3} = \frac{2}{3}(1+a) \Rightarrow \log_2^b = 2(1+a) = 2+2a$

$\Rightarrow a = \log_2^3 \Rightarrow \log_2^b = 2 + 2\log_2^3 = \log_2^4 + \log_2^9 = \log_2^{36} \Rightarrow b = 36$ (۱)

$2b - 8 = 2(36) - 8 = 72 - 8 = 100 \Rightarrow \log^{100} = 2$

۱۰- مجموع ریشه‌ها: $-\frac{b}{2a} = \frac{b}{4a}$ معکوس $\Rightarrow \frac{4a}{b} = \log 4 \Rightarrow 2\log 2 = \frac{4a}{b}$

a و b وسط‌ها $\Rightarrow (\frac{1}{\sqrt[8]{2}})^{\frac{4}{a}} = \frac{1}{2}$ (۰)

$\log_n^m = a \rightarrow m = n^a \qquad \log_{mn}^{m^r n} = b \rightarrow \log_{n^{ra} \times n}^{n^{ra \times n}} = b$ (11

$\rightarrow \frac{ra+1}{a+1} = b \qquad b = r - \frac{1}{a+1} \xrightarrow{a>0} [b] = [r^-] = 1$

$\log_a^a = A \qquad \frac{r}{A} + \frac{1}{r}A = r \rightarrow A^r - rA + r = 0 \rightarrow A = r$ (12

$r = \log_a^q \rightarrow q = a^r \rightarrow a \begin{cases} a = r \checkmark \\ a = -r \times \end{cases}$

(13 $\log^{\omega} = ./V \qquad \log^{q} = ./\Lambda \qquad \log^{1\omega} = 1./1 \qquad \log^{\frac{\omega}{r}} = ./\mu$

$\rightarrow ./\mu a^r + ./\Lambda a - 1./1 \rightarrow \begin{cases} a_1 = 1 \\ a_r = \frac{-1./1}{./r} \end{cases} \rightarrow a_1 - a_r = \frac{1r}{\mu}$

(14 $\frac{\log_r^{\omega} + \log_r^r}{\log_r^{v} + \log_r^r} = \frac{r+1}{r./\Lambda+1} = \frac{1\omega}{19}$

$\frac{\log_\mu^q}{\log_\mu^{1\omega}} = \frac{\log_\mu^r + \log_\mu^\mu}{\log_r^{\omega} + \log_r^\mu} = \frac{\frac{1}{1./q} + 1}{1./\omega + 1} = \frac{1\mu}{r.} = ./q\omega$ (15

$\frac{\log^{1\Lambda}}{\log^{\Lambda}} \Rightarrow \frac{\log^{\mu}}{\log^{r}} = \frac{r^{m}-1}{r} \qquad \log_r^{1r} = 1 + \log_r^{\mu} = 1 + \frac{1}{r}\left(\frac{r^{m}-1}{r}\right) = \frac{r^{m}+\mu}{r}$ (17

$\frac{\log^{\mu}}{r\log^{r}}$

9

$$a=\frac{b+c}{2} \rightarrow b=2a-c \rightarrow \frac{1}{x_1+2x_2}=\frac{1}{S}=\frac{2a}{b}=\frac{2a}{2a-c}=2\log^2$$

$$\rightarrow \frac{2a}{2a-c}=\log^2 \xrightarrow{\text{عکس}} \frac{2a-c}{2a}=\log_2^{10} \rightarrow 1-\log_2^{10}=\frac{c}{2a} \rightarrow 2-\log_2^{100}=\frac{c}{a} \quad (1)$$

$$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}}=\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}}=\left(2^{\frac{c}{a}}\right)^{-\frac{1}{8}} \xrightarrow{(1)} \left(2^{2-\log_2^{100}}\right)^{-\frac{1}{8}}$$

$$\left(\frac{2^2}{2\log_2^{100}}\right)^{\frac{-1}{8}}=\left(\frac{4}{100}\right)^{\frac{-1}{8}}=(25)^{\frac{1}{8}}=\sqrt[4]{5}$$