۱- تمرین $\log_n^m = a$ ، $\log_{mn}^{m^2n} = b$ است اگر $a>0$ باشد حاصل $[b]$ ؟

$$\frac{\log_n^{m^2n}}{\log_n^{mn}} = \frac{(\log_n^m + \log_n^m + \log_n^n)}{(\log_n^m + \log_n^n)} = \frac{2a+1}{a+1} = 1 + \frac{1}{a+1} = b$$

$$[b] = \left[1 + \frac{1}{a+1}\right] = 1$$

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۲- دامنه عبارت‌های زیر

ب) $y = \dfrac{\log_4 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$

$\begin{array}{ccccc} & -1 & & 2 & \\ + & \times & - & \times & + \end{array}$

$x^2 - 1 \geq 0 \rightarrow \begin{array}{ccccc} & -1 & & 1 & \\ + & \bullet & - & \bullet & + \end{array}$

$D_f \; (-\infty, -1) \cup (2, +\infty)$

الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}}^x}}$

$\log_{\frac{1}{2}}^x \rightarrow (x>0) \rightarrow \log_{\frac{1}{2}}^x > 0 \rightarrow (x<1)$

$D_f = (0, 1)$

۳- $2\log_x^a + \underbrace{\log_a^{\sqrt{x}}}_{} = 2$. $x = 9$ مقدار a ؟

$\dfrac{1}{\log_{\sqrt{x}}^a} \qquad \dfrac{1}{2\log_x^a} \Rightarrow 2\log_x^a + \dfrac{1}{2\log_x^a} = 2 \rightarrow 2t + \dfrac{1}{2t} = 2 \xrightarrow{\times 2t}$

$\log_{\sqrt{x}}^a = 2\log_x^a$

$4t^2 - 4t + 1 = 0 \rightarrow (2t-1)^2 \rightarrow t = \frac{1}{2} \rightarrow \log_9^a = \frac{1}{2} \rightarrow \boxed{a=3}$

۴- $\log 2 \simeq 0/3$ ، $\log 3 \simeq 0/48$ آنگاه ریشه‌های معادله $\log\frac{5}{3} x^2 + \log^9 x - \log^{15}$

$(\underbrace{\log 5}_{} - \log 3) x^2 + (\log 3 + \log 3)\, x - (\log 5 + \log 3)$

$\log\frac{10}{2} = (\underbrace{1 - \log 2}_{})$

$1 - 0/3 = 0/7 \rightarrow (0/7 - 0/48) x^2 + 0/96\, x - (0/7 + 0/48) \rightarrow \frac{3}{10} x^2 + \frac{8}{10} x - 1/1$

$3x^2 + 8x - 11 = 0 \rightarrow \dfrac{\sqrt{\Delta}}{|a|} = \dfrac{\sqrt{64 - 4(-33)}}{3} = \dfrac{\sqrt{196}}{3} = \dfrac{14}{3}$

۵- $\log_{14}^{10} = \dfrac{1}{\log_{10}^{14}} = \dfrac{1}{(\log_{10}^7 + \log_{10}^2)}$

$\dfrac{\log_{10}^7}{\log_{10}^2} = 2/8 \xrightarrow{\text{طبق تست بعدی}} \dfrac{\log 7}{\frac{1}{3}} = 2/8 \rightarrow \log 7 = \dfrac{14}{15}$ $\quad \dfrac{\log_{10}^2}{\log_{10}^5} = \dfrac{1}{2} \rightarrow \underbrace{2\log_{10}^2}_{} = \log 5$

$2(1 - \log 5) = \log 5$

$2 - 2\log 5 \rightarrow \log 5 = \frac{2}{3}$

$\rightarrow \log 2 = \frac{1}{3}$

$\dfrac{1}{\frac{14}{15} + \frac{5}{15}} = \dfrac{1}{\frac{19}{15}} = \boxed{\dfrac{15}{19}}$

۶ – $\log_3 5 = 1.5$ و $\log_2 3 = 1.6$ ← $\log_{15} 6$ ؟

$\frac{\log 3}{\log 2} = 1.6$ $\qquad \frac{\log 5}{\log 3} = \frac{3}{2}$

$1.6\log 2 = \log 3 \xrightarrow{\times 3} 4.8\log 2 = 2 - 2\log 2 \to 6.8\log 2 = 2 \to \log 2 = \frac{1}{3.4}$

$\cancel{4\log}\ 2(1-\log 2) = 3\log 3$

$\log_{15} 6 = \frac{\log 6}{\log 15} = \frac{\log 2 + \log 3}{\log 5 + \log 3} \to \frac{\log 2 + 1.6\log 2}{1 - \log 2 + 1.6\log 2} \to \frac{2.6\log 2}{1 + 0.6\log 2}$ $\qquad \frac{\frac{26}{34}\times\frac{10}{10}}{1+\frac{6}{34}\times\frac{10}{10}}$

$\frac{\frac{13}{17}}{1+\frac{3}{17}} = \frac{13}{20}$

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۷ – $\log_8 18 = m$ حاصل $\log_4 12$

$\frac{1}{3}\log_2 18 \to \frac{\log 3 + \log 3 + \log 2}{\log 2} = 3m \to \frac{2\log 3}{\log 2} + 1 = 3m \to \frac{\log 3}{\log 2} = \frac{3m-1}{2}$

$\frac{\log 2 + \log 2 + \log 3}{2\log 2} \to \underbrace{\frac{1}{2} + \frac{1}{2}}_{1} + \underbrace{\frac{\log 3}{2\log 2}}_{\frac{3m-1}{4}} \to \frac{3m-1}{4} + 1 = \frac{3m+3}{4}$

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۸ – اگر $\underbrace{\left(\frac{4}{10}\right)^{2x-1}}_{\left(\frac{2}{5}\right)^{2x-1}} = \underbrace{\left(\frac{125}{8}\right)^{x^2}}_{\left(\frac{5}{2}\right)^{3x^2}}$ باشد $\log_8 (9x+1)$ ؟

$3x^2 = -(2x-1) \to 3x^2 + 2x - 1 = 0$

$\to -1 \to$ غ ق ق

$\to \frac{1}{3} \to \log_8 9(\frac{1}{3}) + 1 = \log_8 4 = \frac{2}{3}\log_2 2 \to \frac{2}{3}$

گرچه خود $\frac{1}{3}$ هم عبارت $2x-1$ را منفی می‌کند

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۹ – اگر $\log_2 3 = a$ و $\log_8 b = \frac{2}{3}(1+a)$ باشد مقدار $\log(3b-8)$ ؟

$\frac{1}{3}\log_2 b = \frac{2}{3}(1+a) \to \log_2 b = 2(\log_2 2 + \log_2 3) \to \log_2 b = 2\log_2 6 = \log_2 36$

$b = 36 \to \log_{10} 100 = 2$

$\sqrt[4]{5}$ جواب بعدی

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۱۰ – عکس مجموع ریشه‌های معادله درجه دوم $-4ax^2 + bx + \frac{1}{2}c$ برابر $\log 4$ و a واسطه حسابی b و c باشد، مقدار $\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}}$

$\frac{1}{\alpha+\beta} = \frac{1}{\frac{-b}{-4a}} = \frac{1}{\frac{b}{4a}} = \log_{10} 4 = \frac{4a}{b} \to b\log_{10} 4 = 4a \to a = \frac{1}{4}b\log_{10} 4$

$\underbrace{\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}}}_{\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}}} = 2^{-\frac{c}{8a}}$

$\frac{b+c}{2} = a \to b + c = \frac{b}{2}\log_{10} 4$ $\qquad -2\log_2 5$ $\qquad \frac{-2\log_{10} 5}{\log_{10} 2}$

$b + c = b\log_{10} 2 \to b(1 - \log_{10} 5)$

$\left(2^{-\frac{1}{8}}\right)^{-2\log_2 5} \to 2^{\frac{1}{4}\log_2 5} \to 5^{\frac{1}{4}} = \sqrt[4]{5}$ $\qquad b + c = b - b\log_{10} 5 \to c = -b\log_{10} 5 \to \frac{c}{a} = \frac{-b\log_{10} 5}{\frac{1}{4}b\log_{10} 4}$