۱_ اگر $\log_n^m = a$ و $\log_{mn}^{m^2n} = b$ است اگر $a>0$ باشد حاصل $[b]$ ؟

$$\frac{\log_n^{m^2n}}{\log_n^{mn}} = \frac{(\log_n^m+\log_n^m+\log_n^n)}{(\log_n^m+\log_n^n)} = \frac{2a+1}{a+1} = 1+\frac{1}{a+1} = b$$

۲۰

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$$[b] = \left[1+\frac{1}{a+1}\right] = 1$$

۲_ دامنه عبارت های زیر

الف) $y=\sqrt{\dfrac{x}{\log_{\frac12}^x}}$

$\log_{\frac12}^x \to x>0 \to \log_{\frac12}^x > 0 \to x<1$

$D_f=(0,1)$

ب) $y=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$

$x^2-x-2>0 \to (x-2)(x+1)>0$

	-1		2	
+	×	−	×	+

$x^2-1\geq 0 \to$

	-1		1	
+	•	−	•	+

$D_f: (-\infty,-1)\cup(2,+\infty)$

۳_ $2\log_x^a + \underbrace{\log_a^{\sqrt x}} = 2$ $\quad x=9$ مقدار a ؟

$$\frac{1}{\log_{\sqrt x}^a = 2\log_x^a} \quad \frac{1}{2\log_x^a} \Rightarrow 2\log_x^a + \frac{1}{2\log_x^a} = 2 \to 2t+\frac{1}{2t} = 2 \xrightarrow{\times 2t}$$

$$4t^2-4t+1=0 \to (2t-1)^2 \to t=\frac12 \to \log_9^a = \frac12 \to \boxed{a=3}$$

۴_ $\log 2 \simeq 0/3$ $\quad \log 3 \simeq 0/48$ $\quad$ اختلاف ریشه‌های معادله $\log\frac{5}{3}x^2+\log 9\, x-\log 15$

$$(\underbrace{\log 5}-\log 3)x^2 + (\log 3+\log 3)x - (\log 5+\log 3)$$

$\log\frac{10}{2} = (\underbrace{1-\log 2})$

$1-0/3 = 0/7 \to (0/7-0/4)x^2 + 0/8\,x - (0/7+0/4) \to \frac{3}{10}x^2+\frac{8}{10}x-1/1$

$$3x^2+8x-11=0 \to \frac{\sqrt\Delta}{|a|} = \frac{\sqrt{64-4(-33)}}{3} = \frac{\sqrt{196}}{3} = \frac{14}{3}$$

۵_ $\log_{14}^{10} = \dfrac{1}{\log_{10}^{14}} = \dfrac{1}{(\log_{10}^7+\log_{10}^2)}$

$$\frac{\log_{10}^7}{\log_{10}^2} = 2/8 \xrightarrow{\text{طبق تست بعدی}} \frac{\log 7}{\frac13} = 2/8 \to \log 7 = \frac{14}{15}$$

$$\frac{\log_{10}^2}{\log_{10}^5} = \frac12 \to \underbrace{2\log_{10}^2} = \log 5$$

$2(1-\log 5) = \log 5$

$2-2\log 5 \to \log 5 = \frac23$

$\to \log 2 = \frac13$

$$\frac{1}{\frac{14}{15}+\frac{5}{15}} = \frac{1}{\frac{19}{15}} = \boxed{\frac{15}{19}}$$

۶ـ $\log_3 5 = 1/5$ و $\log_2 3 = 1/6$ ← $\log_{15} 6$ ؟

$\frac{\log 3}{\log 2} = 1/6$ $\qquad \frac{\log 5}{\log 3} = \frac{3}{2}$

$1/6\log 2 = \log 3 \xrightarrow{\times 3} 4/8\log 2 = 2 - 2\log \to 6/8\log 2 = 2 \to \log 2 = \frac{1}{3/4}$

$\cancel{4\log}\ 2(1-\log 2) = 3\log 3$

(۲)

$\log_{15} 6 = \frac{\log 6}{\log 15} = \frac{\log 2 + \log 3}{\log 5 + \log 3} \to \frac{\log 2 + 1/6\log 2}{1 - \log 2 + 1/6\log 2} \to \frac{2/6\log 2}{1 + 0/6\log 2}$ $\qquad \frac{\frac{\cancel{26}^{13}}{\cancel{10}}\times\frac{\cancel{10}}{\cancel{34}_{17}}}{1+\frac{6}{\cancel{10}}\times\frac{\cancel{10}}{34}_{17}}$

$\frac{\frac{13}{17}}{1+\frac{3}{17}} = \frac{13}{20}$

۷ـ $\log_8 18 = m$ حاصل $\log_4 12$

$\frac{1}{3}\log_2 18 \to \frac{\log 3 + \log 3 + \log 2}{\log 2} = 3m \to \frac{2\log 3}{\log 2} + 1 = 3m \to \frac{\log 3}{\log 2} = \frac{3m-1}{2}$

$\frac{\log 2 + \log 2 + \log 3}{2\log 2} \to \underbrace{\frac{1}{2} + \frac{1}{2}}_{1} + \underbrace{\frac{\log 3}{2\log 2}}_{\frac{3m-1}{4}} \to \frac{3m-1}{4} + 1 = \frac{3m+3}{4}$ (۲)

۸ـ اگر $\underbrace{\left(\frac{4}{10}\right)^{2x-1}}_{\left(\frac{2}{5}\right)^{2x-1}} = \left(\frac{125}{8}\right)^{x^2}$ باشد $\log_8 (9x+1)$ ؟

(۱) $\left(\frac{5}{2}\right)^{3x^2}$

$3x^2 = -(2x-1) \to 3x^2 + 2x - 1 = 0$

$\to (-1) \to$ غ ق ق

$\to \frac{1}{3} \to \log_8 9\left(\frac{1}{3}\right) + 1 = \log_8 4 = \frac{2}{3}\log_2 2 \to \frac{2}{3}$

توجه خود $\frac{1}{3}$ هم عبارت $2x-1$ را منفی می‌کند

۹ـ اگر $\log_2 3 = a$ ، $\log_8 b = \frac{2}{3}(1+a)$ باشد مقدار $\log(3b-8)$ ؟

$\frac{1}{3}\log_2 b = \frac{2}{3}(1+a) \to \log_2 b = 2(\log_2 2 + \log_2 3) \to \log_2 b = 2\log 6 = \log_2 36$

$b = 36 \to \log_{10} 100 = 2$

$\sqrt[4]{5}$ جدا بحث کنیم

۱۰ـ عکس مجموع ریشه‌های معادله درجه دوم $-4ax^2 + bx + \frac{1}{2}c$ برابر $\log 4$ و a واسطه حسابی b و c باشد مقدار $\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}}$

$\frac{1}{\alpha+\beta} = \frac{1}{\frac{-b}{-4a}} = \frac{1}{\frac{b}{4a}} = \log_{10} 4 = \frac{4a}{b} \to b\log_{10} 4 = 4a \to a = \frac{1}{4}b\log_{10} 4$ (۲)

$-2\log_2 5$ $\qquad -\frac{2\log_{10} 5}{\log_{10} 2}$

$\frac{b+c}{2} = a \to b + c = \frac{b}{2}\log_{10} 4$

$\left(2^{-\frac{1}{8}}\right)^{-\log_2 5} \to 2^{\frac{1}{8}\log_2 5} \to 5^{\frac{1}{8}} = \sqrt[8]{5}$

$b + c = b\log_{10} 2 \to b(1 - \log_{10} 5)$

$b + c = b - b\log_{10} 5 \to c = -b\log_{10} 5 \to \frac{c}{a} = \frac{-b\log_{10} 5}{\frac{1}{4}b\log_{10} 2}$

$\underbrace{\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}}}_{\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}}} = 2^{-\frac{c}{8a}}$

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