نام و نام خانوادگی: سانیا جاوری — مقطع: یازدهم B دختر

(۱) $x^2+y^2-4x+6y+k=0 \Rightarrow (x-1)^2+(y+3)^2=k+11 \Rightarrow k+11=0 \to k=-11$

$x^2-4x+2+y^2+6y+9=k+11$

(۲) $x=a \Rightarrow a^2+4a=2a+a+2 \to a^2+a-2=0 \to (a+2)(a-1)=0 \to a=-2 \times \quad a=1 \checkmark$

$f(1)=(1)^2+4(1)=5$

(۳) $x+1 \le -2 \to (-\infty,-3] \cup (1) \to D_f=(-\infty,-3] \cup [1,+\infty)$

$x+1 \ge 2 \to [1,+\infty)$ (۲)

$(1) \cup (2) \cap [-3,-1]=\{-3\} \Rightarrow 2(-3)^2-b(a)+2(-3)=18-b(a)-6$

$24=a+b$

(۴) $y=\sqrt{12-a}+\sqrt{b-4} \Rightarrow \begin{cases} 12-a \ge 0 \to 12 \ge a \\ b-4 \ge 0 \to b \ge 4 \end{cases} \quad \frac{b}{a}=\frac{24}{12}=\frac{1}{2}$

(۵) $\frac{x+1}{|x-3|} \ge 0 \to |x-3| \ne 0 \to x-3 \ne 0 \to x \ne 3$

$x+1 \ge 0 \to x \ge -1 \to D_f=[-1,+\infty)-\{3\}$ (1)

الف) $\frac{4-x}{x^2+2x+5} \ge 0 \to x^2+2x+5 \to \Delta=(2)^2-4(1)(5)=4-20=-16 \to \Delta<0 \to a>0$ (همیشه +)

$4-x \ge 0 \to 4 \ge x \Rightarrow x \le 4$ (2) $\to (1) \cap (2)=[-1,4]-\{3\} \to D_f=[-1,3) \cup (3,4]$

ب) $\sqrt{[x]-4}+\sqrt{2-[x]} \to [x]-4 \ge 0 \to [x] \ge 4$ (I)

$\to 2-[x] \ge 0 \to [x] \le 2$ (II) $\cap \Rightarrow$ ✗

(۶) الف) $\frac{x^2-x-6}{(x^2-4)(x^2-9)} \ge 0 \Rightarrow \frac{(x-3)(x+2)}{(x-2)(x+2)(x-3)(x+3)}$

	-3	-2	2	3
	+	−	+	−

$D_f=(-\infty,-3) \cup (2,3) \cup (3,+\infty)$

ب) $\frac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6} \ge 0 \to (x-1)(x-3)(x+2)$

$x=1 \quad x=3 \quad x=-2$

$(x-2)(x+1)(x+3)$

$x=2 \quad x=-1 \quad x=-3$

	-3	-2	-1	1	2	3
	+	−	+	−	+	−

$D_f=(-\infty,-3) \cup [-2,-1) \cup [1,2) \cup [3,+\infty)$

(۷) الف) $\sqrt{[-x]-2} \Rightarrow [-x]-2 \ge 0 \to [-x] \ge 2 \to -x \ge 2 \to x \le -2$

$D_f=(-\infty,-2]$

$\frac{\omega x+1}{[x]^2-2[x]-3} \geq 0 \Rightarrow D_f = [-1,0) \cup [3,4)$

$([x]-3)([x]+1)$

$[x] \geq 3 \Rightarrow x \in [3,4)$ $\quad [x] = -1 \Rightarrow x \in [-1,0)$

(۷)

الف) $\frac{\cot x+1}{\tan x+1}$ $\begin{cases} \tan x \neq -1 \rightarrow x \neq k\pi - \frac{\pi}{4} \\ \sin x \neq 0 \rightarrow k\pi \\ \cos x \neq k\pi + \frac{\pi}{2} \end{cases}$ $D_f = R - \{k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{2}, k\pi\}$

ب) $\sqrt{1-4\sin^2 x}$

$\rightarrow 1-4\sin^2 x \geq 0 \rightarrow -4\sin^2 x \geq -1 \rightarrow 4\sin^2 x \leq 1$

$\sin^2 x \leq \frac{1}{4} \rightarrow |\sin x| \leq \frac{1}{2} \rightarrow$

$D_f = [k\pi - \frac{\pi}{6}, k\pi + \frac{\pi}{6}]$

(۹)

الف) $y = \sqrt{1 - \log_{\frac{1}{2}}^{x-1}} \rightarrow 1 - \log_{\frac{1}{2}}^{x-1} \geq 0 \rightarrow 1 \geq \log_{\frac{1}{2}}^{x-1} \rightarrow x-1 \leq \frac{1}{2} \rightarrow x \leq \frac{3}{2}$ (I)

$\rightarrow x-1 > 0 \rightarrow x > 1$ (II) $\Rightarrow$ (I) $\cap$ (II) $= (1, \frac{3}{2}]$

ب) $\frac{\sqrt{x^2-x}}{1-\log_4^{x^2-3x}}$ $\rightarrow \log_4^{x(x-3)} \neq 1$

$x^2 - 3x \neq 4$

$x^2 - 3x - 4 \neq 0$

$(x-4)(x+1) \neq 0$ $\quad x \neq 4$, $x \neq -1$

$x^2 - x \geq 0$

$x(x-1) \geq 0$

x		0		1	
$x(x-1)$	$+$	0	$-$	0	$+$

$x^2 - 3x > 0$

x		0		3	
	$+$	0	$-$	0	$+$

$\rightarrow (-\infty, 0) \cup (3, +\infty)$

$\Rightarrow D_f = (-\infty, -1) \cup (-1, 0) \cup (3, 4) \cup (4, +\infty)$

(۱۰)

الف) $\sqrt{\frac{4x-x^2}{x^2-8}}$ $\rightarrow 4x - x^2 \geq 3 \rightarrow -x^2 + 4x - 3 \geq 0 \rightarrow x^2 - 4x + 3 \leq 0$

$(x-3)(x-1)$

$x \leq 3$ $\quad x \geq 1$

x		1		3	
	$+$	0	$-$	0	$+$

$D_f = [1, 3]$

ب) $\frac{3x+5}{2x+4} \in W$

$3x + 5 = 2xw + 4w$

$\Rightarrow 3x - 2xw = 4w - 5$

$\Rightarrow \{x \mid x = \frac{4k-5}{3-2k}, k \in W\}$