توابع و خواص آن‌ها

مینا ستوده تألیف شماره ۲ یازدهم دخترانه 13 ۱۶,۷۵

1. $2x^2-4x+y^2+6y=-k \to 2x^2-4x+y^2+6y+2+9=-k+11 \to (2x-2)^2+(y+3)^2=-k+11 \Rightarrow -k+11=0 \to k=11$

تابع غیرتهی

۳ ✓

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2. $f(x)\begin{cases} x^2+4x & x\geq a \\ 2x+a+2 & x<a \end{cases} \xRightarrow{x=a} a^2+4a=\underbrace{2a+a}_{3a}+2 \to a^2+a-2=0$

$(a+2)(a-1)=0 \begin{cases} a=1 ✓ \\ a=-2 ✗ \end{cases}$

۳ ✓

مطابق شرط ذکر شده $a>0$ چون تنها مقادیر + a قابل قبول بوده بنابراین ۲- غ ق است $\Leftarrow a=1$

$f(1) \xrightarrow[1\leq 1]{1\geq 1} \begin{cases} x^2+4x \xrightarrow{x=1} 1+4=5 ✓ \\ 2x+1+2 \xrightarrow{x=1} 2+1+2=5 ✓ \end{cases}$

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3. $f(x)\begin{cases} 2x^2-b & |x+1|\geq 2 \\ a+2x & -3\leq x\leq -1 \end{cases} \to \begin{cases} x+1\leq -2 \to x\leq -3 \\ x+1\geq 2 \to x\geq 1 \end{cases}$

$-3 \qquad 1$

۴ ✓

$\xrightarrow{x=-3} 2(9)-b=a+2(-3) \to 18-b=a-6 \;,\; a+b=18+6=24$

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4. $\sqrt{6x-a}+\sqrt{b-3x} \to \begin{cases} 6x-a\geq 0 \to x\geq \frac{a}{6} \\ b-3x\geq 0 \to x\geq \frac{-b}{-3} \to x\leq \frac{b}{3} \end{cases}$

$\frac{b}{3}$

دامنه تابع تک نقطه‌ای است $\Rightarrow \frac{b}{3}=\frac{a}{6}=2 \begin{cases} \frac{b}{3}=2 \to b=6 \\ \frac{a}{6}=2 \to a=12 \end{cases}$

۲ ✓

$\Rightarrow \frac{b}{a}=\frac{6}{12}=\frac{1}{2}$

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5. الف

$\frac{x+1}{|x-3|}\geq 0 \quad x+1 \to -1 \quad |x-3| \to 3$

x		-1		3	
$x+1$	$-$	0	$+$		$+$
$\|x-3\|$	$+$		$+$		$+$

ج ن

$\to [-1,3)\cup(3,+\infty)$

$\frac{6-x}{x^2+2x+4}\geq 0 \Rightarrow 6-x\geq 0 \to x\leq 6$

$\to b^2-4ac \to 4-4(1)(4) \;,\; \Delta<0$

ریشه ندارد همواره برقرار

$\cap$

$-1 \quad 3 \quad 6$

✓

$[-1,3)\cup(3,6]$

5-

ب

اشتراک $\longrightarrow$ $D_f = \varnothing$

$[x]-4 \geq 0 \to [x] \geq 4 \to x \geq 4$

$2-[x] \geq 0 \to [x] \leq 2 \quad x < 3$

1.5

الف

6- $\dfrac{x^2-x-6}{x^4-13x^2+36} \geq 0 \quad \to x^2-x-6 \quad (x-3)(x+2) \begin{cases} x=-2 \\ x=3 \end{cases}$

$\xrightarrow{x^2=t} t^2-13t+36 \quad (t-4)(t-9) \begin{cases} t=4 \to x=\pm 2 \\ t=9 \to x=\pm 3 \end{cases}$

x		-3		-2		2		3	
x^2-x-6	$+$		$-$		$-$		$+$		$+$

$\to (-\infty,-3) \cup (2,3) \cup (3,+\infty)$

ب

① $6-5-2+1=0$ جمع ضرایب $\Rightarrow$

$x^3-2x^2-5x+6 \;\big|\; \boxed{x-1}$, $\boxed{x^2-x-6}$

$-x^3+x^2$

$-x^2-5x+6$

$+x^2-x$

$-6x+6$

$+6x-6$

0

$\dfrac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}$

جمع ضرایب توان فرد = زوج توان

② $\dfrac{2-6}{-4} = \dfrac{-5+1}{-4} \Rightarrow$

$x^3+2x^2-5x-6 \;\big|\; \boxed{x+1}$, $\boxed{x^2+x-6}$

$-x^3-x^2$

x^2-5x-6

$-x^2-x$

$-6x-6$

$+6x+6$

0

③ $\dfrac{\overset{1}{(x-1)}(x^2-x-6)}{\underset{-1}{(x+1)}(x^2+x-6)} \geq 0$

$x^2-x-6=0 \to (\underset{3}{x-3})(\underset{-2}{x+2})$

$x^2+x-6=0 \to (\underset{-3}{x+3})(\underset{2}{x-2})$

x		-3		-2		-1		1		2		3	
$x-1$	$-$		$-$		$-$		$-$		$+$		$+$		$+$
x^2-x-6	$+$		$+$		$-$		$-$		$-$		$-$		$+$
$x+1$	$-$		$-$		$-$		$+$		$+$		$+$		$+$
x^2+x-6	$+$		$-$		$-$		$-$		$-$		$+$		$+$
	$+$		$-$		$+$		$-$		$+$		$-$		$+$

$(-\infty,-3) \cup (-2,-1) \cup [1,2) \cup [3,+\infty)$

6

الف

7. $\sqrt{[-x]-2} \to [-x]-2 \geq 0 \quad [-x] \geq 2 \to D_f = [-2,+\infty)$

$x \leq -2$

1.5

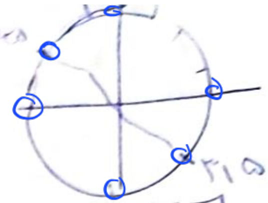
ب

$\dfrac{5x^2+6}{[x]^2-2[x]-3} \to [x]^2-2[x]-3 \neq 0 \xrightarrow{[x]=t} t^2-2t-3=0$

$\to (t+1)(t-3)=0 \begin{cases} t=-1 \to [x]=-1 \to -1 \leq x < 0 \\ t=3 \to [x]=3 \to 3 \leq x < 4 \end{cases}$

$D_f = R - \{[-1,0) \cup [3,4)\}$

8.



الف

$$\frac{\cot x+1}{\tan x+1} \rightarrow \tan x+1 \neq 0 \rightarrow \tan x \neq -1 \quad \left(\frac{\sin}{\cos}\right)$$

$$R-\left[k\pi+\frac{3\pi}{4}\right]$$

$\sin x \neq 0$

$$\cos x \neq 0 \quad R-\left[k\pi+\frac{\pi}{2}\right]$$

$$1 \cap R-\left\{k\pi+\frac{3\pi}{4}, k\pi+\frac{\pi}{2}\right\}$$

ب

$$\sqrt{1-4\sin^2 x} \rightarrow 1-4\sin^2 x \geq 0 \rightarrow 4\sin^2 x \leq 1 \rightarrow \sin^2 x \leq \frac{1}{4}$$

$$-\frac{1}{2} \leq \sin x \leq +\frac{1}{2} \quad x \rightarrow \left[k\pi-\frac{\pi}{6}, k\pi+\frac{\pi}{6}\right]$$

۱,۵

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9. الف

$$\sqrt{1-\log_{\frac{1}{2}}^{x-1}} \rightarrow 1-\log_{\frac{1}{2}}^{x-1} \geq 0 \rightarrow 1 \geq \log_{\frac{1}{2}}^{x-1} \rightarrow \log_{\frac{1}{2}}^{x-1} \leq 1 \rightarrow x-1 \geq \frac{1}{2}$$

$$x \geq 1+\frac{1}{2} = \frac{3}{2}$$

$$\hookrightarrow x-1>0 \rightarrow x>1 \quad \cap \quad x \geq \frac{3}{2}$$

$$\left(1, \frac{3}{2}\right] \qquad D_f = \left[\frac{3}{2}, +\infty\right)$$

1 ، 1,5

۱,۲۵

ب

$$\frac{\sqrt{x^2-x}}{1-\log_4^{x^2-3x}} \rightarrow x^2-x \geq 0 \rightarrow x(x-1) \geq 0 \rightarrow (-\infty,0] \cup [1,+\infty) \quad (1)$$

$$\hookrightarrow x^2-3x>0 \rightarrow x(x-3)>0 \rightarrow (-\infty,0) \cup (3,+\infty) \quad (2)$$

$$1-\log_4^{x^2-3x} \neq 0 \rightarrow \log_4^{x^2-3x} \neq 1 \rightarrow x^2-3x \neq 4 \rightarrow x^2-3x-4 \neq 0 \rightarrow (x-4)(x+1) \rightarrow x \neq 4, \; x \neq -1 \quad (3)$$

$$1 \cap 2 \cap 3 \Rightarrow (-\infty,-1) \cup (-1,0) \cup (3,+\infty) - \{4\}$$

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10. الف

$$\sqrt{2^{4x-x^2}-8} \rightarrow 2^{4x-x^2}-8 \geq 0 \rightarrow 2^{4x-x^2} \geq 2^3 \rightarrow 4x-x^2 \geq 3 \rightarrow -x^2+4x-3 \geq 0$$

$$\rightarrow -(x^2-4x+3) \geq 0 \rightarrow x^2-4x+3 \leq 0 \rightarrow (x-1)(x-3) \quad x=1, \; x=3$$

x		1		3	
x^2-4x+3	+	0	−	0	+

$$\rightarrow [1,3]$$

۱

ب

$$\left(\frac{3x+5}{2x+4}\right)! \rightarrow \frac{3x+5}{2x+4} \geq 0$$

x		-2		$-\frac{5}{3}$	
	−		−		+
	−		+		+
	+		−		+

$$D_f: (-\infty,-2) \cup \left[-\frac{5}{3}, +\infty\right)$$

$$D_f = \left\{x \mid x = \frac{-4k+5}{2k-3}, \; k \in W\right\}$$