

$$\begin{aligned} 2x^2 + y^2 - 4x + 4y + k &= 0 \\ 2x^2 - 4x + \square + y^2 + 4y + 0 &= 0 \\ 2x^2 - 4x + \square + y^2 + 4y + \textcircled{9} &= 0 \\ (\sqrt{2}x - \sqrt{2})^2 + (y + 2)^2 &= 0 \end{aligned}$$

$$\begin{aligned} 0 + \square &= k \\ \downarrow \quad \downarrow \\ 9 + 2 &= 11 = k \end{aligned}$$

✓ ۵

$$f(x) = \begin{cases} x^2 + 4x, & x \geq a \rightarrow a = x \rightarrow a^2 + 4a \\ 2x + a + 2, & x \leq a \rightarrow a = x \rightarrow 2a + a + 2 \end{cases}$$

بهمسانی  
 اندک  
 $a^2 + 4a = 2a + a + 2$   
 $\rightarrow a^2 + a - 2 = 0$   
 $\rightarrow (a+2)(a-1) = 0$

ترتیب  
 $a=1$   
 $a=-2$

$$f(x) = \begin{cases} x^2 + 4x, & x \geq 1 \rightarrow x=1 \rightarrow 1^2 + 4(1) = 5 = f(1) \\ 2x + 3, & x \leq 1 \rightarrow x=1 \rightarrow 2(1) + 3 = 5 = f(1) \end{cases}$$

✓ ۴

$a > 0$  پس غرض

$$f(x) = \begin{cases} 2x^2 - b, & |x+1| \geq 2 \rightarrow x = -3 \rightarrow 2(-3)^2 - b = 11 - b \\ a + 2x, & -3 \leq x \leq -1 \rightarrow x = -3 \rightarrow a + 2(-3) = -4 + a \end{cases}$$

$$11 - b = -4 + a$$

$$11 + 4 = a + b \rightarrow a + b = 15$$

✓ ۵

$$y = \sqrt{4x - a} + \sqrt{b - 3x}$$

$$\begin{cases} 4x - a \geq 0 \rightarrow 4x \geq a \rightarrow x \geq \frac{a}{4} \\ b - 3x \geq 0 \rightarrow b \geq 3x \rightarrow \frac{b}{3} \geq x \end{cases}$$

$$\frac{b}{3} = \frac{a}{4} = \frac{1}{2}$$

$$y = b, a = 12 \leftarrow 2 = \frac{a}{4} = \frac{b}{3}$$

✓ ۵

$$f(x) = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+5}}$$

$$|x-3| \neq 0 \rightarrow x \neq 3 \rightarrow D_1 = \mathbb{R} - \{3\}$$

$$x+1 \geq 0 \rightarrow x \geq -1 \rightarrow D_2 = [-1, +\infty)$$

$$\frac{4-x}{x^2+2x+5} \geq 0 \rightarrow \frac{4}{x^2+2x+5} \geq 0 \rightarrow D_3 = (-\infty, 4]$$

$$\Delta = b^2 - 4ac \rightarrow 4 - 4(1)(5) = -12$$

$$D_T = D_1 \cap D_2 \cap D_3 = [-1, 4] - \{3\}$$

$$f(x) = \sqrt{[x] - 4} + \sqrt{2 - [x]}$$

$$[x] - 4 \geq 0 \rightarrow [x] \geq 4 \rightarrow x \geq 4$$

$$2 - [x] \geq 0 \rightarrow 2 \geq [x] \rightarrow x < 3$$

۵  $D = \emptyset$

|                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>الف) <math>y = \frac{x^2 - x - 4}{x^2 - 13x^2 + 34} \rightarrow x^2 = 6</math></p> <p><math>\frac{x^2 - x - 4}{x^2 - 13x^2 + 34} \geq 0 \rightarrow \frac{(x-2)(x+2)}{(x^2-6)(x^2-9)} \geq 0</math></p> <p><math>\rightarrow \frac{(x-2)(x+2)}{(x^2-6)(x^2-9)} \geq 0</math></p> <p><math>D = (-\infty, -3) \cup (2, +\infty) \cup \{2\}</math></p>                                                                      | <p>ب) <math>y = \frac{x^2 - 2x^2 - 4x + 4}{x^2 + 2x^2 - 4x - 4} \rightarrow \frac{x^2 - x - 4}{x^2 + 2x^2 - 4x - 4}</math></p> <p><math>\frac{(x-1)(x-3)(x+1)}{(x+1)(x+3)(x-2)} \geq 0 \rightarrow \frac{-x^2 - 2x - 1}{(x+1)(x+3)(x-2)} \geq 0</math></p> <p><math>x^2 - x - 4 = 0 \rightarrow (x-3)(x+2) = 0</math></p> <p><math>x^2 + 2x - 4 = 0 \rightarrow (x+3)(x-2) = 0</math></p> <p><math>D = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)</math></p>                                                                               |
| <p>الف) <math>y = \sqrt{-x} - 2</math></p> <p><math>[-x] - 2 \geq 0 \rightarrow [-x] \geq 2</math></p> <p><math>\rightarrow -x \geq 2 \rightarrow x \leq -2</math></p> <p><math>D = (-\infty, -2]</math></p>                                                                                                                                                                                                                | <p>ب) <math>y = \frac{4x^2 + 4}{[x]^2 - 2[x] - 3}</math></p> <p><math>[x]^2 - 2[x] - 3 \neq 0</math></p> <p><math>a^2 - 2a - 3 \neq 0 \rightarrow (a-3)(a+1) \neq 0</math></p> <p><math>[x] \neq 3 \rightarrow x = [3, 4)</math></p> <p><math>[x] \neq -1 \rightarrow x = [-1, 0)</math></p> <p><math>D = \mathbb{R} \setminus [3, 4) \cup [-1, 0)</math></p>                                                                                                                                                                                             |
| <p>الف) <math>y = \frac{\cot x + 1}{\tan x + 1}</math></p> <p><math>\cot x \neq 0 \rightarrow 0^\circ, 180^\circ</math></p> <p><math>\tan x \neq 0 \rightarrow 90^\circ, 270^\circ</math></p> <p><math>\tan x + 1 \neq 0 \rightarrow \tan x \neq -1 \rightarrow 135^\circ, 225^\circ</math></p> <p><math>D = \mathbb{R} \setminus \left\{ \frac{k\pi}{2}, k\pi - \frac{\pi}{4} \right\}</math></p>                          | <p>ب) <math>y = \sqrt{1 - 4\sin^2 x}</math></p> <p><math>1 - 4\sin^2 x \geq 0 \rightarrow 1 \geq 4\sin^2 x \rightarrow \frac{1}{2} \geq \sin^2 x</math></p> <p><math>\rightarrow \frac{1}{2} \geq \sin x \geq -\frac{1}{2}</math></p> <p><math>D = \left[ k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4} \right]</math></p>                                                                                                                                                                                                                                   |
| <p>الف) <math>y = \sqrt{1 - \log \frac{x-1}{x}}</math> <math>x-1 &gt; 0 \rightarrow x &gt; 1</math> (I)</p> <p><math>1 - \log \frac{x-1}{x} \geq 0 \rightarrow 1 \geq \log \frac{x-1}{x} \rightarrow \frac{1}{x} \leq x-1</math></p> <p><math>\rightarrow \frac{1}{x} + 1 \leq x \rightarrow \frac{x+1}{x} \leq x</math> (II)</p> <p><math>D = \mathbb{I} \cap \mathbb{II} = \left[ \frac{2}{3}, +\infty \right)</math></p> | <p>ب) <math>y = \frac{\sqrt{x^2 - x}}{1 - \log \frac{x^2 - x}{x}}</math></p> <p><math>x^2 - x \geq 0 \rightarrow x(x-1) \geq 0</math></p> <p><math>x^2 - 3x \geq 0 \rightarrow x(x-3) \geq 0</math></p> <p><math>1 - \log \frac{x^2 - x}{x} \neq 0 \rightarrow \log \frac{x^2 - x}{x} \neq 1 \rightarrow \frac{x^2 - x}{x} \neq e</math></p> <p><math>\rightarrow x^2 - 2x - e \neq 0 \rightarrow (x-e)(x+1) \neq 0 \rightarrow x \neq -1</math></p> <p><math>D_T = D_1 \cap D_2 \cap D_3 = (-\infty, 0) \cup (1, +\infty) \setminus \{-1, e\}</math></p> |
| <p>الف) <math>y = \sqrt{x - x^2}</math></p> <p><math>x - x^2 \geq 0 \rightarrow x \geq x^2 \rightarrow x - x^2 \geq 0</math></p> <p><math>x^2 - x + 1 \geq 0 \rightarrow x^2 - x + \frac{1}{4} \geq 0 \rightarrow (x - \frac{1}{2})^2 \geq 0</math></p> <p><math>D = [0, 1]</math></p>                                                                                                                                      | <p>ب) <math>y = \left( \frac{x^2 + d}{x^2 + e} \right)!</math></p> <p><math>\frac{x^2 + d}{x^2 + e} \in \mathbb{W} \rightarrow -\frac{x^2 + d}{x^2 + e} = x</math></p> <p><math>\rightarrow x = \frac{-x^2 + d}{x^2 + e}</math></p> <p><math>D = \left\{ x \mid x = \frac{-x^2 + d}{x^2 + e}, x \in \mathbb{W} \right\}</math></p>                                                                                                                                                                                                                        |