

هبة لياض

Date 14/1/22 No

المادة - 1

(A)

$$2x^2 + y^2 - 4x + 4y + k = 0 \rightarrow 2x^2 - 4x + 2 + y^2 + 4y + 4 = 0 \quad (1)$$

$$\rightarrow k = 11$$

(1,0)

المحل الرئيسي

$$x=a, a^2 + \frac{4}{a} = 2a + a + 1 \rightarrow a^2 + a - 2 = 0 \rightarrow (a+2)(a-1) \quad (2)$$

$$a=1 \Rightarrow f(1) = 1^2 + (1^2 + 1) = 3$$

$$f(x) = \begin{cases} 2x^2 - b & x \geq 1 \\ a + 2x & x \leq -3 \end{cases} \quad x = -1 \rightarrow 11 - b = a - 4 \quad (3)$$

$$-3 \leq x \leq -1 \quad a + b = 14$$

$$x - a \geq 0 \rightarrow x \geq \frac{a}{9} \quad b - 3x \geq 0 \rightarrow x \leq \frac{b}{3} \quad (4)$$

$$\frac{b}{3} = \frac{a}{9} = 2 \rightarrow \frac{b}{a} = \frac{2}{9} = \frac{b}{a} = \frac{1}{4}$$

(5)

$$x \geq 1 \cap x \neq 3 \quad \text{و} \quad x - 2 \geq 0 \rightarrow x \leq 2$$

$$D_f = [1, 2] - \{3\}$$

$$[x] \geq 4 \rightarrow [4, +\infty)$$

$$\rightarrow D_f = \emptyset$$

$$2 - [x] \geq 0 \rightarrow [x] \leq 2 \rightarrow (-\infty, 3)$$

$$(الف) \quad \sqrt{\frac{(x-3)(x+2)}{x^2+13x+42}} \rightarrow x \mid \begin{array}{c} \star \quad \star \\ -3 \quad -2 \quad 2 \quad 3 \\ \hline + \quad - \quad - \quad + \end{array}$$

$$P \mid \begin{array}{c} + \quad - \quad - \quad + \\ \hline + \quad - \quad - \quad + \end{array}$$

(6)

$$x = \pm 3 \quad x = \pm 2 \quad D_f = (-\infty, -3) \cup (2, +\infty) - \{3\}$$

بنا خذ

ب) $\sqrt{\frac{x^3 - 2x^2 - 5x + 6}{x^3 + 2x^2 - 5x - 6}} \div \frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)}$ (2)

$$\begin{array}{r} x^3 - 2x^2 - 5x + 6 \div (x-1) \\ \underline{-x^3 + x^2} \\ -x^2 - 5x + 6 \\ \underline{+x^2 + x} \\ -4x + 6 \\ \underline{+4x - 4} \\ 2 \end{array} \quad \begin{array}{r} x^3 + 2x^2 - 5x - 6 \div (x+1) \\ \underline{-x^3 - x^2} \\ 3x^2 - 5x - 6 \\ \underline{-3x^2 - 3x} \\ -2x - 6 \\ \underline{+2x + 2} \\ -4 \end{array}$$

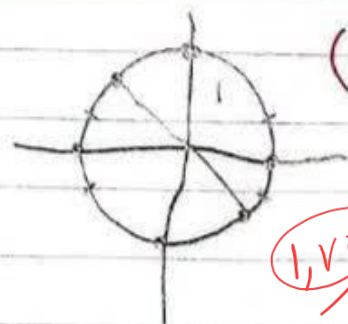
x	-3	-2	-1	1	2	3
P	+	-	+	-	+	-

$\rightarrow D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$

الف) $[-x] - 2 \geq 0 \rightarrow [-x] \geq 2 \rightarrow x \in (-\infty, -2)$ (1, 1, 1, 1)

ب) $[x^2] - 2[x] - 3 = 0 \rightarrow ([x] - 3)([x] + 1)$
 $[x] = 3 \rightarrow [3, 4) \rightarrow \emptyset$
 $[x] = -1 \rightarrow [-1, 0) \rightarrow \emptyset$
 $D_f = \mathbb{R} - \{[5, 6), [-1, 0)\}$

الف) $\frac{\cos x + \sin x}{\sin x} = \frac{\sin x + \cos x}{\cos x}$
 $\sin x \neq 0$
 $\cos x \neq 0$
 $\sin x \neq \cos x$



$D_y = \mathbb{R} - \left\{ k\frac{\pi}{4}, k\pi - \frac{\pi}{4} \right\}$

ب) $y = \sqrt{1 - 4\sin^2 x} \rightarrow (1 - 4\sin^2 x)(1 + 4\sin^2 x) \geq 0$

$1 - 4\sin^2 x \geq 0 \rightarrow \sin^2 x \leq \frac{1}{4} \rightarrow \sin x \leq \frac{1}{2} \cup \sin x \geq -\frac{1}{2}$
 $\rightarrow \left[\frac{\pi}{6}, \frac{5\pi}{6} \right] \cup \left[\frac{7\pi}{6}, \frac{11\pi}{6} \right]$
 $D = \left[k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4} \right]$

(9)

الف) $x-1 > 0$, $\log_{\frac{1}{2}} x-1 \leq 1 \rightarrow x-1 \geq \frac{1}{2} \rightarrow x \geq \frac{3}{2}$

$D_f = [\frac{3}{2}, +\infty)$ ✓

(5)

ب) $x^2 - x \geq 0 \rightarrow x(x-1) \geq 0$ $\begin{array}{c|c|c|c} & 0 & 1 & \\ \hline + & - & + & \end{array}$ $\begin{array}{c|c|c|c} & 0 & 1 & 3 \\ \hline + & - & - & + \end{array} \star$

$\log_{\frac{1}{2}} x^2 - 3x \geq 1 \rightarrow x^2 - 3x \leq 2 \rightarrow x^2 - 3x - 2 \rightarrow (x-4)(x+1)$
 $x^2 - 3x \geq 0 \rightarrow \star$ $x \neq 4, -1$

$\hookrightarrow D_f = (-\infty, 0] \cup (3, +\infty) - \{4, -1\}$

(10)

الف) $2^{3x-x^2} \geq 1 \rightarrow 3x-x^2 \geq 0 \rightarrow x^2-3x \leq 0$

$(x-3)(x-0) \leq 0$ $\begin{array}{c|c|c|c} & 0 & 3 & \\ \hline + & - & + & \end{array} \rightarrow [0, 3] \neq D_f$

(5)

ب) $\left(\frac{3x+5}{2x+4}\right)!$ راه مستقیم $\frac{3x+5}{2x+4} \in \mathbb{N} \rightarrow 3x+5 = 2x(2x+4) + 4$
 راه تقابلی $\frac{3x+5}{2x+4} \in \mathbb{N} \rightarrow 3x+5 = 2x(2x+4) + 4$

$\frac{3x+5}{2x+4} \in \mathbb{N} \rightarrow 3x+5 = 2x(2x+4) + 4$

$\hookrightarrow 3x - 2x(2x+4) = 4 - 5 \rightarrow 2x(3-2x-4) = 4-5$

$x = \frac{4-5}{3-2x-4} = \frac{-1}{-x-1}$

جواب نهایی: $\{x \mid x = \frac{-k+5}{2k-3}, k \in \mathbb{N}\}$