

$$\begin{aligned} 2x^2 + y^2 - 4x + 6y + K &= 0 \\ y^2 + 6y + (2x^2 - 4x + K) &= 0 \\ \Delta = 0 &= 36 - 4(1)(2x^2 - 4x + K) = 0 \\ 36 - 8x^2 + 16x - 4K &= 0 \\ -4(2x^2 - 4x + K - 9) &= 0 \end{aligned}$$

$$\begin{aligned} \Delta' &= 16 - 4(2)(K - 9) = 0 \\ 16 - 8K + 72 &= 0 \\ -8K + 88 &= 0 \\ -K + 11 &= 0 \\ \boxed{K = 11} \end{aligned}$$

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$$\begin{aligned} a^2 + 4a &= 2a + a + 2 \\ a^2 + a - 2 &= 0 \\ (a + 2)(a - 1) &= 0 \end{aligned}$$

$$f_{(1)} = 1^2 + 4(1) = \boxed{5}$$

$a = \begin{cases} -2 \times \text{غیر مقبول} \\ \boxed{1} \end{cases}$

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$$\begin{aligned} |x+1| &\geq 2 \\ x+1 &\geq 2, -2 \geq x+1 \\ x &\geq 1, x \leq -3 \end{aligned}$$

$$\begin{aligned} 2(-3)^2 - b &= a + 2(-3) \\ 18 + 6 &= a + b \\ \boxed{a + b = 24} \end{aligned}$$

$$\Rightarrow f(x) = \begin{cases} 2x^2 - b & x \in [1, +\infty) \cup (-\infty, -3] \\ a + 2x & -3 \leq x \leq -1 \end{cases}$$

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$$\begin{aligned} 9x - a &\geq 0 & b - 3x &\geq 0 \\ 9x &\geq a & b &\geq 3x \\ x &\geq \frac{a}{9} & x &\leq \frac{b}{3} \end{aligned}$$

$$D = (-\infty, \frac{b}{3}] \cap [\frac{a}{9}, +\infty) = \{2\}$$

$$\frac{b}{3} = \frac{a}{9} = 2$$

$$\frac{b}{3} = \frac{a}{9} \quad \frac{b}{a} = \frac{3}{9} = \boxed{\frac{1}{3}}$$

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الف) $|x-3| \neq 0$
 $x \neq 3$

$$\frac{x+1}{|x-3|} > 0$$

$$\begin{aligned} x+1 &> 0 \\ x &> -1 \end{aligned}$$

ب) $[x] - 4 > 0$
 $[x] > 4$
 $x > 4$

$$\begin{aligned} 2 - [x] &> 0 \\ [x] &< 2 \\ x &< 2 \\ (-\infty, 2) \end{aligned}$$

$$\begin{aligned} x^2 + 2x + 4 &\neq 0 \\ (x+2)^2 &\neq 0 \\ x &\neq -2 \end{aligned}$$

$$\frac{9-x}{(x+2)^2} > 0$$

$$\begin{aligned} 9-x &> 0 \\ x &\leq 9 \end{aligned}$$

$$D = (-\infty, 9] \cap (-\infty, -2) - \{-2, 3\} = \boxed{[-1, 9] - \{3\}}$$

$$D = [4, +\infty) \cap (-\infty, 2) = \boxed{\emptyset}$$

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الف) $x^3 - 13x^2 + 49x = 0$
 $(x^2 - 9)(x - 7) = 0$
 $x^2 = 9 \Rightarrow x = 3, -3$
 $x = 3, -3, 7$ $\cup$
 $\frac{x^2 - 9}{x^2 - 13x^2 + 49x} > 0$
 $\frac{(x-3)(x+3)}{(x-7)(x+7)(x-7)(x+7)} > 0$

x	-3	-2	3	7
y	$+$	$+$	$-$	$+$

$$D = (-\infty, -2) \cup (2, +\infty) - \{7\}$$

ب) $(x^2 - x - 6)(x - 1) > 0$ $(x+3)(x-2)(x-1) = 0$
 $(x^2 + x - 6)(x+1) > 0$ $x = -3, 2, -1, 0 =$
 $(x-3)(x+2)(x-1)$
 $(x+3)(x-2)(x+1) > 0$

x	-3	-2	-1	1	2	3
y	$+$	$+$	$-$	$-$	$+$	$+$

$$D = (-\infty, -2) \cup (-2, -1) \cup (1, 2) \cup (2, +\infty)$$

الف) $[-x] - 2 > 0$ $[-x] > 2$ $-x > 2$ $x < -2$ $D = [-\infty, -2]$

ب) $[x]^2 - 2[x] - 3 = 0$

$$t = [x]$$

$$t^2 - 2t - 3 = 0$$

$$(t-3)(t+1) = 0$$

$$t = 3 \text{ or } -1$$

$$+ \begin{cases} t = -1 \Rightarrow [x] = -1 \Rightarrow x \in [-1, 0) \\ t = 3 \Rightarrow [x] = 3 \Rightarrow x \in [3, 4) \end{cases}$$

$$\Rightarrow D = \mathbb{R} - ([1, 2) \cup [2, 3])$$

الف) $\tan x + 1 = 0$

$$\tan x = -1$$

$$x = \frac{\sqrt{2}\pi}{4}$$

$$D = \mathbb{R} - \left\{ \frac{\sqrt{2}\pi}{4} \right\}$$

ب) $1 - \sin^2 x > 0$

$$(1 - \sin x)(1 + \sin x) = 0$$

$$\sin x = \begin{cases} \frac{1}{2} \Rightarrow x = \frac{\pi}{6} \\ -\frac{1}{2} \Rightarrow x = \frac{11\pi}{6} \end{cases}$$

$$D = \mathbb{R} - \left\{ \frac{\pi}{6}, \frac{11\pi}{6} \right\}$$

الف) $x - 1 > 0$ $1 - \log_{\frac{1}{2}} x > 0$ $\log_{\frac{1}{2}} x \leq 1$ $x - 1 \leq \frac{1}{2}$ $x \leq \frac{3}{2}$ $D = (1, \frac{3}{2}]$

ب) $x^2 - x > 0$

$$\frac{x}{x} \frac{1}{1} \frac{0}{0} \frac{1}{0} \frac{1}{0}$$

$$(-\infty, 0) \cup (1, +\infty)$$

$$x^2 - 3x > 0$$

$$\frac{x}{x} \frac{0}{1} \frac{0}{0} \frac{3}{0} \frac{1}{0}$$

$$(-\infty, 0) \cup (3, +\infty)$$

$$1 - \log_{\frac{1}{2}} x^2 - 3x \neq 0$$

$$\log_{\frac{1}{2}} x^2 - 3x = 1$$

$$x^2 - 3x - 1 = 0$$

$$\mathbb{R} - \{-1, 4\}$$

$$D = (-\infty, 0) \cup (3, +\infty) - \{-1, 4\}$$

الف) $\sqrt{x-x^2} - 1 > 0$

$$\sqrt{x-x^2} > 1$$

$$x - x^2 = 1$$

$$x^2 - x + 1 = 0$$

$$(x-2)(x-1) = 0$$

$$\frac{x}{x} \frac{1}{0} \frac{2}{0} \frac{1}{0}$$

$$D = [1, 2]$$

ب) $\frac{3x+5}{2x+7} \in \mathbb{W}$

$$3x+5 = 2xw+7w$$

$$3x-2xw = 7w-5$$

$$x = \frac{7w-5}{3-2w}$$

$$D = \left\{ x \mid x = \frac{7w-5}{3-2w}, w \in \mathbb{W} \right\}$$