

$$\begin{aligned}
 & 2a^2 + y^2 - 2a + 4y + k = 0 \quad \text{در دو طرف } y^2 + 4y + m = (y + \sqrt{m})^2, m = 4 \\
 & y^2 + 4y + 4 = (y + 2)^2 \quad 2a^2 - 2a = 2(a^2 - a) \quad a^2 - 2a + 1 = (a - 1)^2 \\
 & a = 1 \rightarrow a^2 - 2a + 1 = (a - 1)^2 \\
 & \Rightarrow (y + 2)^2 + 2(a - 1)^2 = 0 \rightarrow \begin{cases} y + 2 = 0 \\ a - 1 = 0 \end{cases} \rightarrow \begin{cases} y = -2 \\ a = 1 \end{cases} \rightarrow 2a^2 + 1^2 \times 2 = 4 + 2 = 6
 \end{aligned}$$

$$\begin{aligned}
 f(a) &= \begin{cases} a^2 + 2a & ; a \geq a \\ 2a + a + 2 & ; a \leq a \end{cases} \xrightarrow{a=a} a^2 + 2a = -a^2 + 2a = 2a + 2 \rightarrow a + a - 2 = 0 \rightarrow \\
 & (a + 2)(a - 1) = 0 \\
 f(a) &= \begin{cases} a^2 + 2a & ; a \geq 1 \\ 2a + a + 2 & ; a \leq 1 \end{cases} \rightarrow f(1) = 1 + 2 = 3 \\
 & a = 1 \rightarrow 1 - 2 = -1 \rightarrow a = 1
 \end{aligned}$$

$$\begin{aligned}
 f(a) &= \begin{cases} 2a^2 - b & ; |a+1| \geq 2 \\ a + 2a & ; -2 \leq a \leq -1 \end{cases} \rightarrow a = -2 \quad 2 \times 9 - b = 9 - 4 = 18 - b, a = 4 \\
 & |a+1| \geq 2 \rightarrow a+1 \geq 2 \rightarrow a \geq 1 \\
 & \quad \quad \quad \rightarrow a+1 \leq -2 \rightarrow a \leq -3 \\
 & a + b = 18 + 4 = 22
 \end{aligned}$$

$$\begin{aligned}
 y &= \sqrt{4a - a} + \sqrt{b - 2a} \\
 4a - a &\geq 0 \rightarrow \begin{cases} 4a - a \geq 0 \\ b - 2a \geq 0 \end{cases} \rightarrow \begin{cases} 3a \geq 0 \\ b \geq 2a \end{cases} \rightarrow \begin{cases} a \geq 0 \\ b \geq 2a \end{cases} \\
 \frac{b}{a} &= \frac{4}{12} = \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{الف) } y &= \sqrt{\frac{a+1}{|a-2|}} + \sqrt{\frac{4-a}{2a^2+2a+2}} \\
 \frac{a+1}{|a-2|} &\geq 0 \rightarrow \begin{cases} a+1 \geq 0 \\ |a-2| > 0 \end{cases} \rightarrow \begin{cases} a \geq -1 \\ a \neq 2 \end{cases} \\
 4-a &\geq 0 \rightarrow a \leq 4 \\
 \text{ب) } y &= \sqrt{[a]-4} + \sqrt{2-[a]} \\
 [a]-4 &\geq 0 \rightarrow [a] \geq 4 \rightarrow a \geq 4 \quad \text{①} \\
 2-[a] &\geq 0 \rightarrow [a] \leq 2 \rightarrow a \leq 3 \quad \text{②} \\
 \text{① و ②} &\rightarrow \text{برای هیچ قدری از } a \text{ برقرار نیست}
 \end{aligned}$$

الف) $y = \sqrt{\frac{x^2 - x - 4}{x^2 - 13x + 4}} \geq 0$ $(x-3)(x+2) \geq 0$ $(x^2 - 9)(x^2 - 13x + 4) \geq 0$ $(x-3)(x+2)(x-1)(x+4) \geq 0$ $D_f = \mathbb{R} - \{1, 4\}$ $D_f = (-\infty, -4) \cup (-1, 1) \cup (1, 3) \cup (3, +\infty)$	ب) $y = \sqrt{\frac{x^2 - 13x + 4}{x^2 - x - 4}} \geq 0$ $(x-1)(x-4) \geq 0$ $(x^2 - 13x + 4)(x^2 - x - 4) \geq 0$ $(x-1)(x-4)(x-1)(x+4) \geq 0$ $D_f = \mathbb{R} - \{1, 4\}$ $D_f = (-\infty, -4) \cup (-1, 1) \cup (1, 3) \cup (3, +\infty)$
الف) $y = \sqrt{[x] - 2} \geq 0$ $[x] \geq 2 \rightarrow -x \geq 2 \rightarrow x \leq -2$ $D_f = (-\infty, -2]$	ب) $y = \sqrt{2[x] + 4}$ $[x] \geq -2 \rightarrow x \geq -2$ $D_f = [-2, +\infty)$
الف) $y = \frac{\cot x + 1}{\tan x + 1}$ $\tan x \neq -1$ $\cot x = \frac{\cos x}{\sin x}$ $\tan x = \frac{\sin x}{\cos x}$ $D_f = \mathbb{R} - \{k\pi + \frac{\pi}{4}, k\pi + \frac{5\pi}{4}\}$	ب) $y = \sqrt{1 - \sin^2 x}$ $\sin^2 x \leq 1 \rightarrow \sin x \leq 1$ $-\frac{1}{\sqrt{2}} \leq \sin x \leq \frac{1}{\sqrt{2}}$ $D_f = [k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4}]$
الف) $y = \sqrt{1 - \log \frac{x-1}{x}} \geq 0$ $x > 1$ $1 - \log \frac{x-1}{x} \geq 0 \rightarrow \log \frac{x-1}{x} \leq 1$ $x-1 \geq \frac{1}{x} \rightarrow x \geq \frac{1}{x-1}$ $D_f = [\frac{1}{x-1}, +\infty)$	ب) $y = \sqrt{x^2 - x} \geq 0$ $x^2 - x \geq 0 \rightarrow x(x-1) \geq 0$ $x \leq 0$ or $x \geq 1$ $D_f = (-\infty, 0] \cup [1, +\infty)$
الف) $y = \sqrt{r^2 \tan^2 \alpha - r^2} \geq 0$ $r^2 \tan^2 \alpha \geq r^2 \rightarrow \tan^2 \alpha \geq 1$ $\tan \alpha \geq 1$ or $\tan \alpha \leq -1$ $D_f = [1, +\infty) \cup (-\infty, -1]$	ب) $y = \frac{(r^2 + d)}{r^2 + k}$ $r^2 + d = k^2 + d \rightarrow r^2 - k^2 = d - d = 0$ $r^2 - k^2 = 0 \rightarrow (r-k)(r+k) = 0$ $r = k$ or $r = -k$ $D_f = \{r \mid r = \frac{k^2 - d}{r - k}, k \in \mathbb{W}\}$