

$$2x^2 + y^2 - 4x + 4y + k = 0 \quad ()^2 + ()^2 = 0$$

$$2(x+1)^2 + (y+2)^2 = 0$$

$$2(x^2 - 2x + 1) + (y^2 + 4y + 4) = 0$$

$$2(x-1)^2 + (y+2)^2 = 0$$

$$\rightarrow 2x^2 - 4x + \underline{2} + y^2 + 4y + \underline{4} = 0 \quad k = 2 + 4 = 6$$

$$f(x) = \begin{cases} x^2 + 2x; & x \geq a \\ 2x + a + 2; & x < a \end{cases}$$

$$\begin{aligned} x=a & \rightarrow a^2 + 2a = 2a + a + 2 \\ \rightarrow a^2 + a - 2 = 0 \\ (a+2)(a-1) = 0 \end{aligned}$$

$$\begin{cases} a = -2 \\ a = 1 \end{cases}$$

$$f(1) = 1^2 + 2 \cdot 1 = 1 + 2 = 3$$

$$f(x) = \begin{cases} 2x^2 - b & |x+1| \geq 2 \rightarrow \begin{cases} x+1 \geq 2 \rightarrow x \geq 1 \\ x+1 \leq -2 \rightarrow x \leq -3 \end{cases} \\ a + 2x & -3 \leq x \leq -1 \end{cases}$$

$$\xrightarrow[n=-3]{\text{در } x=-3} 2(-3)^2 - b = a + 2(-3) \rightarrow 18 - b = a - 6 \Rightarrow a + b = 24$$

$$y = \sqrt{9x - a} + \sqrt{b - 3x} \quad D_y = \{x\}$$

$$\sqrt{9(1) - a} + \sqrt{b - 3(1)} = 0 \rightarrow \begin{cases} 9 - a = 0 \Rightarrow a = 9 \\ b - 3 = 0 \Rightarrow b = 3 \end{cases}$$

$$\frac{b}{a} = \frac{3}{9} = \frac{1}{3}$$

$$\text{الف) } y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{9-x}{x^2+2x+3}}$$

$$\begin{aligned} x+1 &\geq 0 \\ x &\geq -1 \quad \textcircled{I} \end{aligned}$$

$$\begin{aligned} 9-x &\geq 0 \\ x &\leq 9 \quad \textcircled{II} \end{aligned}$$

$$\Delta = 4 - 16 = -12 < 0$$

$$\textcircled{I} \cap \textcircled{II} \rightarrow [1, 9] - \{3\}$$

$$\text{ب) } y = \sqrt{[x] - 4} + \sqrt{4 - [x]}$$

$$\begin{aligned} [x] - 4 &\geq 0 \\ [x] &\geq 4 \rightarrow [4, +\infty) \quad \textcircled{I} \end{aligned}$$

$$\begin{aligned} 4 - [x] &\geq 0 \\ [x] &\leq 4 \rightarrow (-\infty, 4] \quad \textcircled{II} \end{aligned}$$

$$\textcircled{I} \cap \textcircled{II} \rightarrow \emptyset$$

$$a) y = \frac{(x-3)(x+3)}{(x-3)(x+3)(x-1)(x+1)} \geq 0 \quad \rightarrow x=3, -3$$

$$D_y = (-\infty, -3) \cup (-1, 3) \cup (3, +\infty)$$

$$\rightarrow y = \frac{x^3 - 3x^2 - 3x + 9}{x^4 + x^3 - 4x^2 - 9} \geq 0 \quad \frac{(x-1)(x-3)(x+1)}{(x+1)(x+3)(x-1)} \geq 0$$

$$D = (-\infty, -3) \cup [-1, -1) \cup [1, 3) \cup [3, +\infty) \quad \rightarrow x = -1, -3, 1, 3$$

$$a) y = \sqrt{[-x]-2} \quad [-x]-2 \geq 0 \quad [-x] \geq 2 \quad -x \geq 2 \quad x \leq -2$$

$$D_y = (-\infty, -2]$$

$$\rightarrow y = \frac{2x^2 + 3}{[x]^2 - 2[x] - 3} \geq 0 \quad \frac{2x^2 + 3}{\frac{[x]^2 - 2[x] - 3}{x}} \geq 0 \quad [x] = 3 \rightarrow [3, 3) \quad [x] = -1 \rightarrow [-1, 0)$$

$$x^2 - 2x - 3 \geq 0 \quad (x+1)(x-3) \geq 0 \quad \begin{cases} x=3 \\ x=-1 \end{cases} \quad D_y = \mathbb{R} - [3, 3) \cup [-1, 0)$$

$$D_y = (-\infty, -1) \cup [-1, 3) \cup [3, +\infty) \cup (-\infty, -1)$$

$$a) y = \frac{\cot x + 1}{\tan x + 1} \rightarrow \frac{\cos x}{\sin x} \rightarrow \sin x \neq 0$$

$$\mathbb{R} - \left\{ \frac{k\pi}{2}, k\pi + \frac{\pi}{2} \right\} \subseteq \mathbb{R} - \left\{ \frac{k\pi}{2}, k\pi - \frac{\pi}{2} \right\}$$

$$\rightarrow y = \sqrt{1 - \sin^2 x} \rightarrow 1 - \sin^2 x \geq 0 \quad \sin^2 x \leq \frac{1}{4} \rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$$

$$1) y = \sqrt{1 - \log \frac{x-1}{x}} \quad 1 - \log \frac{x-1}{x} \geq 0 \quad \log \frac{x-1}{x} \leq 1 \rightarrow x-1 \geq \frac{1}{x} \quad x \geq \frac{x}{x}$$

$$D_y = [\frac{x}{x}, +\infty)$$

$$2) y = \frac{\sqrt{x^2 - x}}{1 - \log \frac{x^2 - x}{x}} \rightarrow x^2 - x \geq 0 \quad x(x-1) \geq 0 \quad \frac{x(x-1)}{x} \geq 0 \quad x(x-1) \geq 0$$

$$\log \frac{x^2 - x}{x} \neq 1 \rightarrow x^2 - x \neq x \rightarrow x^2 - 2x \neq 0 \quad D_y = (-\infty, 0) \cup (2, +\infty) - \{1, 2\}$$

$$a) y = \sqrt{x^2 - x^2 - 1} \rightarrow x^2 - x^2 - 1 \geq 0 \quad x^2 - x^2 \geq 1$$

$$x^2 - x^2 \geq 1 \rightarrow -x^2 + x^2 - 1 \geq 0 \quad x^2 - x^2 + 1 \leq 0 \quad \frac{1}{x} - \frac{1}{x} + 1 \leq 0$$

$$D_y = [1, 3]$$

$$\rightarrow y = \left(\frac{x^2 + 4}{x^2 + 1} \right)! \quad \frac{x^2 + 4}{x^2 + 1} \in \mathbb{N} \quad x = - \frac{f(w) - 4}{x^2 - 1}$$

$$D_f = \{x \mid x = \frac{-5k + 4}{2k - 1}, k \in \mathbb{N}\}$$