

$$2x^2 + y^2 - 8x + 4y + k = 0$$

$$(x\sqrt{2} - \sqrt{2})^2 + (y-2)^2 = 0$$

$$2x^2 + 2 - 4x + y^2 + 4 - 4y = 0$$

$$k = 4 + 2 = 11 \rightarrow \boxed{k=11}$$

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$$f(x) = \begin{cases} x^2 + 8x & ; x \geq a \\ 2x + a + 2 & ; x \leq a \end{cases}$$

هنگامی که $x=a$ دو ضابطه برابرند

$$x=a \rightarrow a^2 + 8a = 2a + a + 2$$

$$a^2 + a - 2 = 0 \rightarrow a = -2, 1 \xrightarrow{a \geq 0} a=1 \Rightarrow f(x) = \begin{cases} x^2 + 8x & ; x \geq 1 \\ 2x + 3 & ; x \leq 1 \end{cases} \xrightarrow{x=1} 2(1) + 3 = 5 \rightarrow \boxed{f(1)=5}$$

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$$f(x) = \begin{cases} 2x^2 - b & ; |x+1| \geq 2 \rightarrow |x+1| \geq 2 \begin{cases} x+1 \geq 2 \rightarrow x \geq 1 \\ x+1 \leq -2 \rightarrow x \leq -3 \end{cases} \\ a + 2x & ; -3 \leq x \leq -1 \end{cases}$$

در نقطه $x=-3$ دو ضابطه برابرند

$$x=-3 \rightarrow a - 4 = 12 - b$$

$$a + b = 12 + 4 \rightarrow \boxed{a+b=16}$$

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$$y = \sqrt{\frac{4x-a}{4}} + \sqrt{\frac{b-3x}{3}}$$

$x = \frac{a}{4}$ $x = \frac{b}{3}$

$$x=2 \rightarrow \frac{a}{4} = \frac{b}{3} = 2 \rightarrow a=8, b=6$$

$$\boxed{\frac{a}{b} = \frac{4}{12} = \frac{1}{3}}$$

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$$y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+5}}$$

$$\frac{x+1}{|x-3|} \geq 0$$

$\xrightarrow{x \neq 3}$

$$\frac{-1}{-1+3} \geq 0$$

$$\textcircled{1} \cap \textcircled{5} = D_f$$

$$D_f = [-1, 3) \cup (3, 4]$$

$$\frac{4-x}{(x+2)^2} \geq 0$$

$\xrightarrow{x \neq -2}$

$$\frac{-2^2}{+3+4} \geq 0$$

$$y = \sqrt{[x] - 2} + \sqrt{2 - [x]}$$

$$[x] - 2 \geq 0 \rightarrow [x] \geq 2 \rightarrow x \in [2, +\infty) \textcircled{I}$$

$$2 - [x] \geq 0 \rightarrow [x] \leq 2 \rightarrow x \in (-\infty, 3) \textcircled{II}$$

$$\textcircled{I} \cap \textcircled{II} = \emptyset$$

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الف) $y = \sqrt{\frac{x^2 - x - 4}{x^2 - 13x^2 + 34}}$ $(x^2 - 3)(x^2 + 2) \geq 0$ $(x^2 - 4)(x^2 - 9)$ $\rightarrow \pm 2 \quad \rightarrow \pm 3$ $D_f = (-\infty, 2) \cup (2, 3) \cup (3, +\infty)$	ب) $y = \sqrt{\frac{x^3 - 2x^2 - 5x + 4}{x^3 + 2x - 5x - 4}}$ $x^3 - 2x^2 - 5x + 4 \rightarrow \div x - 1 : (x - 1)(x^2 - x - 4)$ $x^3 + 2x - 5x - 4 \rightarrow \div x + 1 : (x + 1)(x^2 - x - 4)$ $D_f = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$	٦
الف) $y = \sqrt{[-x] - 2} \rightarrow [-x] - 2 \geq 0 \rightarrow [-x] \geq 2 \rightarrow x \leq -2 \rightarrow D_f = (-\infty, -2]$ ب) $y = \frac{5x^2 + 4}{[x]^2 - 2[x] - 3} \rightarrow \neq 0$ $[x]^2 - 2[x] - 3 \neq 0 \rightarrow ([x] + 1)([x] - 3) \neq 0$ $x \in [-1, 0) = 0 \quad x \in [3, 4) = 0$ $D_f = (-\infty, -1) \cup [0, 3) \cup [4, +\infty)$		٧
الف) $y = \frac{\cot x + 1}{\tan x + 1}$ $D_f = R - \left\{ \frac{k\pi}{2}, k\pi + \frac{3\pi}{4} \right\}$ $\cot = \frac{1}{\tan}$ تقريب زائد $\tan = -1$ معكوس تربيع زائد ب) $y = \sqrt{1 - \varepsilon \sin^2 x} \rightarrow 1 - \varepsilon \sin^2 x \geq 0 \rightarrow \varepsilon \sin^2 x \leq 1 \rightarrow \sin^2 x \leq \frac{1}{\varepsilon} \rightarrow -\frac{1}{\sqrt{\varepsilon}} \leq \sin x \leq \frac{1}{\sqrt{\varepsilon}}$ $D_f = \left[k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4} \right]$		٨
الف) $y = \sqrt{1 - \log_{\frac{1}{2}} x - 1} \rightarrow 1 - \log_{\frac{1}{2}} x - 1 \geq 0 \rightarrow -\log_{\frac{1}{2}} x \geq 0 \rightarrow \log_{\frac{1}{2}} x \leq 0 \rightarrow x - 1 \geq \frac{1}{2} \rightarrow x \geq \frac{3}{2} \rightarrow I \cap II \rightarrow D_f = \left[\frac{3}{2}, +\infty \right)$ ب) $y = \frac{\sqrt{x^2 - x}}{1 - \log_{\frac{1}{2}} x} \rightarrow$ ① $x^2 - x \geq 0 \rightarrow x(x - 1) \geq 0 \rightarrow \frac{0}{+} \frac{0}{+} \frac{1}{-} \frac{1}{+}$ ② $1 - \log_{\frac{1}{2}} x \neq 0 \rightarrow \log_{\frac{1}{2}} x \neq 1 \rightarrow x^2 - 2x \neq 4$ $x \neq -1, 4$ $D_f = ① \cap ② \cap ③ = (-\infty, -1) \cup (-1, 0) \cup (3, 4) \cup (4, +\infty)$		٩
الف) $y = \sqrt{\frac{\varepsilon n - n^2}{\varepsilon n - n^2 - 1}} \rightarrow \frac{\varepsilon n - n^2}{\varepsilon n - n^2 - 1} \geq 0 \rightarrow \frac{\varepsilon n - n^2}{\varepsilon n - n^2 - 1} \geq 0 \rightarrow \varepsilon n - n^2 - 1 \geq 0 \rightarrow x^2 - \varepsilon n + 1 \leq 0$ $x = 3, 1$ $D_f = [1, 3]$ ب) $y = \left(\frac{r_n + 5}{r_n + 2} \right)!$ $\frac{r_n + 5}{r_n + 2} \in \mathbb{W}$ $D_f = \{x \mid x \in \frac{-\varepsilon k + 5}{r_k - 2}, k \in \mathbb{W}\}$		١٠