(۱) سایا جاوری

$B(۲,۸)$ $A(۶,-۴)$

$M\left(\frac{۶+۲}{۲}, \frac{-۴+۸}{۲}\right) = M(۴,۲)$

$m_{AB} = \frac{۸-(-۴)}{۲-۶} = \frac{۱۲}{-۴} = -۳ \rightarrow m' = \frac{۱}{m} \rightarrow m' = \frac{۱}{۳} \rightarrow y-۲ = \frac{۱}{۳}(x-۴)$

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(۲) ABC ← $C(-۱,۲)$ $B(-۶,۷)$ $A(۳,۱)$

AH, BC

$|BC| = \sqrt{(-۱+۶)^۲+(۲-۷)^۲} = ۵$

$AB, AC \rightarrow S = \frac{۱}{۲}\begin{vmatrix} -۵ & ۲ \\ -۶ & ۱ \end{vmatrix} = \frac{۱}{۲} \times |-۵+۱۲| = ۵$

$S = \frac{۱}{۲} \times BC \times AH \rightarrow AH = ۲$

$BC - AH = ۳$

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(۳) $۴y = ax+۲$ $۲y-x = ۷$

$۲y-x = ۷ \rightarrow y = \frac{۱}{۲}x + \frac{۷}{۲} \rightarrow m_۱ = \frac{۱}{۲}$

$۴y = ax+۲ \rightarrow y = \frac{a}{۴}x + \frac{۱}{۲}$ $m_۲ = \frac{a}{۴}$

$\frac{a}{۴} = \frac{۱}{۲} \rightarrow a = ۲$

$۲y - x - ۷ = ۰$

$d = \frac{|C_۰ - C'_۰|}{\sqrt{A^۲+B^۲}} = \frac{|۱-۷-(-۱)|}{\sqrt{(-۱)^۲+۲^۲}}$

$۴y = ۸x+۲ \rightarrow ۲y - x - ۱ = ۰$

$= \frac{۶}{\sqrt{۵}}$

$r = \frac{۳}{\sqrt{۵}} \rightarrow S = \pi r^۲ = \pi\left(\frac{۳}{\sqrt{۵}}\right)^۲ = \frac{۹\pi}{۵}$

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(۴) $A(۲,۴)$ $B(۶,۱)$ $C(-۱,-۳)$

M و H پای میانه و پای ارتفاع وارد بر AB طول MH

$M\left(\frac{۲+۶}{۲}, \frac{۴+۱}{۲}\right) = (۴,۲۵)$ $m = \frac{۱-۳}{۶-۲} = -۱ \rightarrow y-۳ = -۱(x-۲) \Rightarrow y = -x+۵$

$x+y-۵ = ۰$

$H: \frac{a x_۰ + b y_۰ + c_۰}{\sqrt{a^۲+b^۲}}, \frac{-۱-۳-۵}{۱^۲+۱^۲} = \frac{-۹}{۲}$

$MH = \sqrt{(۴-۳/۵)^۲+(۲-۱,۵)^۲}$

$x = -۱-۱ \times \frac{-۹}{۲} = -۱+\frac{۹}{۲} = \frac{۷}{۲} \Rightarrow H(۳/۵, ۱,۵)$

$MH = \sqrt{(۰/۵)^۲+(۰,۵)^۲}$

$y = -۳-۱ \times \frac{-۹}{۲} = -۳+\frac{۹}{۲} = \frac{۳}{۲}$

$MH = \sqrt{\frac{۱}{۴}+\frac{۱}{۴}} = \sqrt{\frac{۱}{۲}}$

$MH = \frac{\sqrt{۲}}{۲}$

۵) به ازای چه مقدار m، $(m+1)x-(2m-4)y+1=0$ و $(3m-2)x+(5-m)y-7=0$ عمود بر هم

$m_1=-\frac{3m-2}{5-m}$

$m_2=\frac{m+1}{2m-4}$

$m_1\times m_2=-1 \rightarrow -\frac{3m-2}{5-m}\times\frac{m+1}{2m-4}=-1$

$\frac{(3m-2)(m+1)}{(5-m)(2m-4)}=1$

$(3m-2)(m+1)=(5-m)(2m-4)$

$3m^2+m-2=-2m^2+14m-20$

$5m^2-13m+18=0$

$\Delta=(-13)^2-4(5)(18)=169-360=-191$

$\Delta<0 \rightarrow$ مقدار حقیقی برای m وجود ندارد

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۶) $\begin{cases} AB: x+2y=3 \\ AC: y=2x-1 \\ BC: x+y=4 \end{cases}$

میانه $\frac{AM}{AH}=?$

$x+4x-2=3 \rightarrow 5x=5 \rightarrow x_A=1$

$y_A=2(1)-1=1 \rightarrow A(1,1)$

$x+2y=3$

$x+y=4 \rightarrow x=4-y$

$4-y+2y=3 \rightarrow 4+y=3 \rightarrow y_B=-1$

$x_B=5$

$x+(2x-1)=4 \Rightarrow 3x=5 \rightarrow x_C=\frac{5}{3}$

$y_C=4-x_C=4-\frac{5}{3}=\frac{12-5}{3}=\frac{7}{3}$

$x_C=\frac{5}{3}$, $y_C=\frac{7}{3}$

$M=\left(\frac{x_B+x_C}{2},\frac{y_B+y_C}{2}\right)$

$x_M=\frac{5+\frac{5}{3}}{2}=\frac{\frac{15+5}{3}}{2}=\frac{20}{6}=\frac{10}{3}$

$M\left(\frac{10}{3},\frac{2}{3}\right)$

$AM=\sqrt{\left(\frac{10}{3}-1\right)^2+\left(\frac{2}{3}-1\right)^2}=\sqrt{\frac{49}{9}+\frac{1}{9}}=\sqrt{\frac{50}{9}}=\frac{5\sqrt{2}}{3}$

$AH=\frac{|1(1)+1(1)-4|}{\sqrt{1^2+1^2}}=\frac{|1+1-4|}{\sqrt{2}}$

$\frac{2}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\sqrt{2}$

$\frac{AM}{AH}=\frac{5\sqrt{2}}{3\sqrt{2}}=\boxed{\frac{5}{3}}$

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(۷) $m = -\frac{1}{2}$ — $\sqrt{20}$

$y = -\frac{1}{2}x + b \longrightarrow \frac{1}{2}x + y - b = 0 \longrightarrow x + 2y - 2b = 0$

$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} \longrightarrow \sqrt{20} = \frac{|1(0) + 2(0) - 2b|}{\sqrt{1^2 + 2^2}} = \sqrt{20} = \frac{|1 - 2b|}{\sqrt{5}}$

$|-2b| = \sqrt{20} \times \sqrt{5}$

$2b = \sqrt{100}$

$|2b| = 10$

$b = 5$

$x \to 0 \to y = b = 5 \to A(0, 5)$

$y \to 0 \to 0 = -\frac{1}{2}x + 5 \to x = 10$

$B(10, 0)$

$|AB| = \sqrt{(10-0)^2 + (0-5)^2} = \sqrt{100+25} = \sqrt{125} = 5\sqrt{5} \longrightarrow AB = 5\sqrt{5}$

(۸) $my - 4x = 2m$, $y + mx = x + 2 \to y = (1-m)x + 2$ $= \frac{5}{2}$

$y = \frac{4}{m}x + 2$ عرض x ها

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$0 = (1-m)x + 2 \implies x_1 = \frac{-2}{1-m}$

$0 = \frac{4}{m}x + 2 \implies x_2 = -\frac{m}{2}$

قاعده $= |x_1 - x_2| = \left|-\frac{m}{2} + \frac{2}{1-m}\right| \longrightarrow \frac{5}{2} \longrightarrow \left|\frac{m^2 - m + 4}{2(1-m)}\right| = \frac{5}{2}$

یا $\nearrow \frac{m^2 - m + 4}{2(1-m)} = \frac{5}{2} \longrightarrow m^2 - m + 4 = 5 - 5m \longrightarrow m = -2 \pm \sqrt{5}$

$\searrow \frac{m^2 - m + 4}{2(1-m)} = -\frac{5}{2} \longrightarrow m^2 - m + 4 = -5 + 5m \longrightarrow m^2 - 6m + 9 = 0$

$(m-3)^2 = 0 \longrightarrow m = 3$

$m_1 \times m_2 \times m_3 = (-1)(3) = -3$

DATE $\begin{cases} x' = 2x_M - x = 4 - x \\ \sout{y' = 2y_M - y = -4 - y} \end{cases}$ (9) SUBJECT :

$M(2,-2)$ $y : 2x - 3$

$y = 2x - 3$

$y = -4 - y' , \quad x = 4 - x'$

$-4 - y' = 2(4 - x') - 3$

$-4 - y' = 8 - 2x' - 3 \rightarrow \boxed{y' = 2x' - 9}$

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(10) $A(1,2) \rightarrow y = x + 5$ $A'(a,b)$ $2b - a = ?$

$y = x + 5 \longrightarrow x - y + 5 = 0$

$A = 1 \quad B = -1 \quad C = 5$

$ax + by + c = (1)(1) + (-1)(2) + 5 = 4$

$a^2 + b^2 = 1 + 1 = 2$

$x' = 1 - \frac{2(1)(4)}{2} = 1 - 4 = -3$

$y' = 2 - \frac{2(-1)(4)}{2} = 2 - (-4) = 6$

$A'(-3,6)$

$a = -3 \quad b = 6$

$2b - a = 2(6) - (-3) = \boxed{12 + 3 = 15}$