

$$x_c = \frac{x_A + x_B}{2} = \frac{1 + (-1)}{2} = 0$$

$$y_c = \frac{y_A + y_B}{2} = \frac{4 + 1}{2} = 2.5$$

$$\Rightarrow c \parallel l$$

$$a_{AB} = \frac{4 - 1}{-1 - 1} = -\frac{3}{2}$$

$$a_d = -\frac{1}{a_{AB}} = \frac{2}{3}$$

$$y - y_c = a_d (x - x_c)$$

$$y - 2.5 = \frac{2}{3} (x - 0) \Rightarrow y = \frac{2}{3}x + 2.5$$

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$$BC = \sqrt{(3 - 1)^2 + (1 - 4)^2} = \sqrt{4 + 9} = \sqrt{13}$$

$$BC \text{ خط } a_{BC} = \frac{1 - 4}{-1 - 3} = -\frac{3}{4}$$

$$AH = \frac{|1 \cdot 1 + \frac{2}{3}(3) - \frac{25}{3}|}{\sqrt{1^2 + (\frac{2}{3})^2}} = \frac{|\frac{3}{3} + \frac{4}{3} - \frac{25}{3}|}{\sqrt{1 + \frac{4}{9}}} = \frac{|\frac{-18}{3}|}{\sqrt{\frac{13}{9}}} = \frac{6}{\frac{\sqrt{13}}{3}} = \frac{18}{\sqrt{13}}$$

$$y - 3 = -\frac{4}{3}(x + 1)$$

$$y = -\frac{4}{3}x + \frac{5}{3} \Rightarrow y + \frac{4}{3}x - \frac{5}{3} = 0$$

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$$BC - AH = \sqrt{13} - \frac{18}{\sqrt{13}} = \frac{13 - 18}{\sqrt{13}} = -\frac{5}{\sqrt{13}}$$

$$\Rightarrow \frac{4}{3} = \frac{a}{1} \Rightarrow a = \frac{4}{3} \Rightarrow 4y - 3x - 5 = 0$$

$$r = \frac{|c' - c|}{\sqrt{a'^2 + b'^2}} = \frac{|-5 - (-1)|}{\sqrt{4^2 + (-3)^2}} = \frac{4}{5} = \text{قطر دایره} \Rightarrow r = \frac{2}{5}$$

$$S_{\text{دایره}} = r^2 \pi = (\frac{2}{5})^2 \times 3.14 = \frac{4}{25} \times 3.14 = \frac{12.56}{25} = 0.5024$$

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$$AB \text{ خط } a_{AB} = \frac{4 - 1}{-1 - 1} = -\frac{3}{2} \Rightarrow y - 1 = -\frac{3}{2}(x - 1) \Rightarrow y = -\frac{3}{2}x + \frac{5}{2}$$

$$CH \text{ خط } a_{CH} = -\frac{1}{a_{AB}} = \frac{2}{3} \Rightarrow y - (-3) = \frac{2}{3}(x - (-1)) \Rightarrow y = \frac{2}{3}x - 2$$

$$H = \text{نقطه تقاطع } CH, AB \Rightarrow \begin{cases} y = -\frac{3}{2}x + \frac{5}{2} \\ y = \frac{2}{3}x - 2 \end{cases} \Rightarrow y = \frac{3}{2} \Rightarrow x_H = \frac{1}{2}$$

$$M = \text{نقطه تقاطع } \Rightarrow x_M = \frac{1 + 1}{2} = 1$$

$$y_M = \frac{3 + 1}{2} = 2 \Rightarrow MH = \sqrt{(\frac{1}{2} - 1)^2 + (2 - \frac{3}{2})^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$$

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$$a = \frac{3m - 2}{m - 5} \quad a' = \frac{m + 1}{2m - 4}$$

$$a = -\frac{1}{a'} \Rightarrow \frac{3m - 2}{m - 5} = \frac{4 - 2m}{m + 1} \Rightarrow 3m^2 + m - 2 = -2m^2 + 14m - 20$$

$$\Delta = 1^2 - 4 \times 5 \times 18 = 1 - 360 = -359$$

$$5m^2 - 13m + 18 = 0$$

از آنجمله مقدار m این دو خط بر هم عمود نیستند $\Rightarrow \Delta < 0$

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$$\begin{cases} y = -x + 1 \\ -y = -x + 1 \\ 0 = -2x + 2 \Rightarrow x = 1 \Rightarrow y = 0 \end{cases} \quad A | 1$$

$$\begin{cases} y = 2x - 1 \\ -y = x - 1 \\ x = 2x - 2 \Rightarrow x = 2 \Rightarrow y = 3 \end{cases} \quad C | 2$$

$$\begin{cases} x = -2y + 1 \\ -x = y - 1 \\ 0 = -y - 1 \Rightarrow y = -1 \Rightarrow x = 2 \end{cases} \quad B | -1$$

$$M = \text{center: } x_M = \frac{2+1}{2} = \frac{3}{2} \Rightarrow AM = \sqrt{\left(\frac{3}{2}-1\right)^2 + \left(\frac{1}{2}-0\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{\sqrt{2}}{2}$$

$$y_M = \frac{-1+1}{2} = 0$$

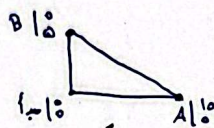
$$\text{BC side: } a_{BC} = \frac{\frac{3}{2}+1}{\frac{3}{2}-2} = -1 \Rightarrow y+1 = -(x-2) \Rightarrow y = -x+1 \Rightarrow y+x-1=0$$

$$AH = \frac{|1x+1x-1|}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad \frac{AM}{AH} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{1}$$

$$\sqrt{20} = 2\sqrt{5} = \frac{|ax_0 + by_0 - y_B|}{\sqrt{a^2 + b^2}} = \frac{|-y_B|}{\sqrt{(-\frac{1}{2})^2 + 1}} = \frac{|-y_B|}{\frac{\sqrt{5}}{2}} \Rightarrow |-y_B| = \Delta \Rightarrow y_B = \Delta$$

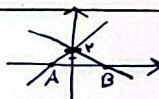
$$y = -\frac{x}{2} + y_B \Rightarrow y = -\frac{x}{2} + \Delta$$

$$0 = -\frac{x_A}{2} + \Delta \Rightarrow x_A = 2\Delta$$



$$AB = \sqrt{\Delta^2 + 0^2} = \sqrt{12\Delta} = \boxed{\Delta\sqrt{3}}$$

$$y = \frac{r}{m}x + r \quad / \quad y = (-m+1)x + r$$



$$S = \frac{\Delta}{r} = \frac{r \times AB}{r} \Rightarrow AB = \frac{\Delta}{r}$$

$$0 = \frac{r}{m}x + r \Rightarrow x = -\frac{m}{r}$$

$$\Rightarrow \textcircled{1} \left(\frac{r}{-m+1} + \frac{m}{r} = \frac{\Delta}{r} \right) \Rightarrow -\frac{r^2}{m^2 - 2m + 1} + m = \Delta - \Delta m$$

$$0 = (-m+1)x + r \Rightarrow x = \frac{r}{1-m}$$

$$\textcircled{2} \left(\frac{-m}{r} + \frac{r}{-m+1} = \frac{\Delta}{r} \right) \Rightarrow (m-2)r = 0 \Rightarrow \underline{m=2}$$

$$\Rightarrow -m + m^2 + r = \Delta - \Delta m \Rightarrow m^2 + r(m-1) = \Delta - \Delta m \Rightarrow m = \frac{\pm\sqrt{20} - r}{r} = \frac{\pm\sqrt{5} - 2}{1}$$

$$\text{مساحت مثلث} = (-2 + \sqrt{5})(-2 - \sqrt{5}) \times 2 = (4 - 5) \times 2 = \boxed{-2}$$

نقطه M روی نیمساز ناصبه دوم و چهارم <=> قرینه خط نسبت به نیمساز ناصبه دوم و چهارم <=> $y = -x$ و $x = -y$

$$-x = r(-y) - r$$

$$-x = -ry - r$$

$$ry = x - r$$

$$\boxed{y = \frac{x}{r} - \frac{r}{r}}$$

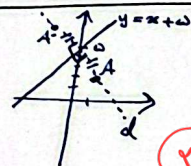
$$\frac{\alpha + r}{r} = 1 \Rightarrow \alpha = r$$

$$\frac{\beta - r}{r} = -1 \Rightarrow \beta = -1$$

$$\left. \begin{matrix} \alpha = r \\ \beta = -1 \end{matrix} \right\} \rightarrow m_{AB} = 1$$

$$y = rx - r$$

$$g = \frac{|r-1-\Delta|}{\sqrt{1^2 + (-1)^2}} = \frac{r}{\sqrt{2}} \Rightarrow \frac{|-x_0 + y_0 - \Delta|}{\sqrt{2}} = \frac{r}{\sqrt{2}} \Rightarrow |y_0 - x_0 - \Delta| = r$$



$$\text{دлина } \Delta \Rightarrow a = -\frac{1}{1} = -1 \Rightarrow y - r = -(x - 1) \Rightarrow y = -x + r$$

$$|-x_0 + r - x_0 - \Delta| = r$$

$$|-2x_0 - r| = r \Rightarrow |-x_0 - 1| = r$$

$$\begin{aligned} -x_0 - 1 = r &\Rightarrow x_0 = -r - 1 \Rightarrow y_0 = r - 1 \Rightarrow \left| \frac{r}{r} \right| \quad A' \text{ نقطه} \\ -x_0 - 1 = -r &\Rightarrow x_0 = 1 \Rightarrow y_0 = r \Rightarrow \left| \frac{1}{r} \right| \quad \text{هكون نقطه} \end{aligned}$$

$$A' \left| \frac{a}{b} \right| \Rightarrow A' \left| \frac{-3}{6} \right| \Rightarrow a = -3, b = 6 \quad r b - a = 12 - (-3) = \boxed{15}$$