DATE / / سانیا جاوری SUBJECT :

① $y = -x^2 + bx + 3$

$y_{max} = -\frac{b^2}{4a} + c \xrightarrow{\Delta} y_{max} = -\frac{b^2}{4(-1)} + 3 = \frac{b^2}{4} + 3$

$x_{max} = -\frac{b}{2a}$

$\frac{b^2}{4} + 3 = 7 \rightarrow \frac{b^2}{4} = 4 \Rightarrow b^2 = 16$

$b = \pm 4$

② الف) $f(x) = x^4 + 2x^2 + 3 \longrightarrow t = x^2 \rightarrow t^2 + 2t + 3 = 0$

$\Delta = 2^2 - 4(1)(3) = 4 - 12 = -8 < 0$

ریشه حقیقی وجود ندارد

ب) $f(x) = (4 - x^2)^2 - 2(4 - x^2) - 15$

$(4 - x^2)^2 = 16 - 8x^2 + x^4 \rightarrow f(x) = x^4 - 8x^2 + 16 - 8 + 2x^2 - 15$

$f(x) = x^4 - 6x^2 - 7 \rightarrow f(x) = 0 \rightarrow x^4 - 6x^2 - 7 = 0$

$y = x^2 \rightarrow y = 7 \rightarrow x^2 = 7 \rightarrow x = \pm\sqrt{7}$

$y^2 - 6y - 7 = 0$

$y = \frac{6 \pm \sqrt{36 + 28}}{2} = \frac{6 \pm \sqrt{64}}{2} = \frac{6 \pm 8}{2}$

$y = 7$ $\quad$ $y = -1$ بدون جواب

DATE / / SUBJECT :- (ریاضیات)

(۲) $4x^2 - 12x + m = 0$

$x_1 = x_2 + 2$

$\begin{cases} x_1 + x_2 = \frac{12}{4} = 3 \\ x_1 x_2 = \frac{m}{4} \end{cases}$

$(x_2+2) + x_2 = 3 \Rightarrow 2x_2 = 1 \Rightarrow x_2 = \frac{1}{2} \Rightarrow x_1 = \frac{5}{2}$

$x_1 x_2 = 2 \Rightarrow \frac{m}{4} = 2 \Rightarrow m = 12$

(۳) $3x^2 - 4x + m - 1 = 0$ $\quad 2\alpha - \beta = 4$

$\begin{cases} \alpha + \beta = 2 \\ \alpha - \beta = 4 \end{cases} \Rightarrow 3\alpha = 6 \Rightarrow \alpha = 2, \beta = 0$

$\alpha\beta = 0 \Rightarrow \frac{m-1}{3} = 0 \Rightarrow m = 1$

(۵) $-mx^2 + 4x + m^2 - 2 = 0$ $\quad \frac{1}{\alpha}, \alpha$

$\alpha + \frac{1}{\alpha} = -\frac{b}{a}, \quad \alpha \cdot \frac{1}{\alpha} = \frac{c}{a}$

$a = -m \quad b = 4 \quad c = m^2 - 2 \longrightarrow \alpha \cdot \frac{1}{\alpha} = 1 = \frac{c}{a} = \frac{m^2-2}{-m} = 1 \rightarrow -m = m^2 - 2$

$m^2 + m - 2 = 0$

غیرقابل قبول $m = -2 \leftarrow$ $\quad \swarrow m = 1$

(۶) $\alpha, \beta \quad x^2 - mx + 2 = 0 \quad \alpha\beta^2 = 4$

$\alpha + \beta = m \quad \alpha\beta = 2$

$\alpha\beta^2 = 4 \longrightarrow (\alpha\beta)\beta = 4 \rightarrow 2\beta = 4 \rightarrow \beta = \frac{4}{2}$

$\alpha = \frac{2}{\beta} = \frac{2}{\frac{4}{2}} = \frac{9}{4} \longrightarrow m = \alpha + \beta = \frac{9}{4} + \frac{4}{2} = \frac{17}{12} = m = \frac{43}{12}$

DATE / / SUBJECT :

(۷) $x^2 - 4x + m = 0$

2α , α

$\begin{cases} \alpha + 2\alpha = 4 \rightarrow 3\alpha = 4 \rightarrow \alpha = 1 \\ \alpha \cdot 2\alpha = 2\alpha^2 = m \\ m = 2(1)^2 = 2 \end{cases}$

(۸) $x^2 - 7x + 2 = 0$

α $\qquad \alpha^2 + \frac{8}{\alpha^2}$ $\qquad \alpha + \frac{2}{\alpha} = 7$

$\alpha^2 + \frac{8}{\alpha^2} = \left(\alpha + \frac{2}{\alpha}\right)^2 - 2 \times 2\left(\alpha + \frac{2}{\alpha}\right) \quad y = \alpha + \frac{2}{\alpha} = 7 \Rightarrow y = 7$

$\hookrightarrow y^2 - 4y = 7^2 - 4(7) = 49 - 28 = 21 \rightarrow \boxed{\alpha^2 + \frac{8}{\alpha^2} = 21}$

(۹) $h(t) = -10t^2 + 50t$

$t_{max} = -\frac{b}{2a} = -\frac{50}{2(-10)} = 2{,}5 \qquad h_{max} = -10(2{,}5)^2 + 50(2{,}5) = -10(6{,}25)$

$+12{,}5$

$h(t) = 0 \rightarrow -10t^2 + 50t = 0 \rightarrow -10t(t-5) = 0 \rightarrow t = 5 \quad h = -62{,}5 + 12{,}5 = 50$

$\boxed{t = 5}$

ب) $(x-1)^2 = 2x + 1$

$y = (x-1)^2$

$y = 2x + 1$

$x^2 - 2x + 1 = 2x + 1 \Rightarrow x^2 - 4x = 0 \rightarrow x(x-4) = 0$

$x = 0$

$\boxed{x = 0, 4}$

ج) $x^2 + 2x = |x+2| \Rightarrow x + 2 \geq 0 \rightarrow x \geq -2$

$x^2 + 2x = x + 2 \rightarrow x^2 + x - 2 = 0 \rightarrow (x+2)(x-1) = 0 \rightarrow \boxed{x = -2, 1}$

$(x+2)(x+1) \overset{=0}{\rightarrow} x = \{-2, -1\}$