نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۱۴ ..... کلاس .....

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$y = -x^2 + bx + 3$

$x_{min} = \frac{-b}{2a} \Rightarrow \frac{b}{2a} \quad y_{max} = -\left(\frac{-b}{2a}\right)^2 + b\left(\frac{b}{2}\right) + 3 \Rightarrow -\frac{b^2}{4} + \frac{2b^2}{4} + 3 \Rightarrow$

$y_{max} = \frac{b^2}{4} + 3 \quad \frac{b^2}{4} + 3 = 7 \quad \frac{b^2}{4} = 4 \quad b^2 = 16 \quad \boxed{b = \pm 4}$

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الف) $f(x) = x^4 + 2x^2 + 3 \quad t = x^2 \quad t^2 + 2t + 3 = 0 \quad \Delta < 0$ ریشه حقیقی ندارد

ب) $f(x) = (4 - x^2)^2 - 2(4 - x^2) - 15 \quad t = 4 - x^2 \quad t^2 - 2t - 15 = 0 \quad (t-5)(t+3) \Rightarrow$

$t = 5 \rightarrow 4 - x^2 = 5 \quad x^2 = -1$ ✗

$t = -3 \rightarrow 4 - x^2 = -3 \quad x^2 = 7 \rightarrow \boxed{x = \pm\sqrt{7}}$

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$4x^2 - 14x + m = 0 \quad \alpha = \beta + 2$

$\alpha + \beta = \frac{-b}{a} \quad 2\beta + 2 = \frac{14}{4} \quad 2\beta = 2 \quad \boxed{\beta = 1} \quad \boxed{\alpha = 3}$

$\alpha \times \beta = \frac{c}{a} \quad \beta^2 + 2\beta = \frac{m}{4} \quad 3 = \frac{m}{4} \quad \boxed{m = 12}$

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$3x^2 - 7x + m - 1 = 0 \quad 2\alpha - \beta = 4 \quad \beta = 2\alpha - 4$

$\alpha + \beta = \frac{7}{3} = 3 \quad \alpha + 2\alpha - 4 = 3 \quad 3\alpha - 4 = 3 \quad \alpha = \frac{7}{3} \quad \boxed{\alpha = 2} \quad \boxed{\beta = 0}$

$\alpha \times \beta = \frac{m-1}{3} \quad \frac{m-1}{3} = 0 \quad m - 1 = 0 \quad \boxed{m = 1}$

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$-mx^2 + 4x + m^2 - 2 = 0$

معکوس $\rightarrow \frac{c}{a} = 1 = \frac{-m^2 - 2}{m} = 1 \quad -m + 2 = m \quad m^2 + m - 2 = 0$

$(m+2)(m-1) = 0 \rightarrow \boxed{m = -2} \quad \boxed{m = 1}$

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$x^2 - mx + 3 = 0 \qquad \alpha\beta^2 = 4$

$\underbrace{\alpha \times \beta}_{3} = \frac{4}{\beta} \qquad \frac{4}{\beta} = 3 \quad \boxed{\beta = \frac{4}{3}} \qquad \alpha = \frac{4}{\frac{16}{9}} = \boxed{\frac{9}{4}}$

$\alpha + \beta = m \qquad m = \frac{9}{4} + \frac{4}{3} = \boxed{\frac{43}{12}}$

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$x^2 - 4x + m = 0$

$\alpha = 3\beta$

$\alpha\beta = m \qquad 3\beta^2 = m \qquad \uparrow \longrightarrow \boxed{m = 3}$

$\underbrace{\alpha}_{3\beta} + \beta = 4 \qquad 4\beta = 4 \quad \boxed{\beta = 1} \quad \boxed{\alpha = 3}$

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$x^2 - 7x + 2 = 0 \qquad \alpha^3 + \frac{8}{\alpha^3} \longrightarrow \alpha^3 + 8\left(\frac{1}{\alpha^3}\right) \longrightarrow$

$\downarrow$

$\alpha^2 - 7\alpha + 2 = 0 \qquad \alpha^2 = 7\alpha - 2 \xrightarrow{\times\alpha} \alpha^3 = 7\alpha^2 - 2\alpha = \boxed{\ }$

$\downarrow$

$\frac{1}{\alpha} = t \qquad \left(\frac{1}{t}\right)^2 - 7\frac{1}{t} + 2 = 0 \Rightarrow 2t^2 - 7t + 1 = 0 \rightarrow t_1 = \frac{2t-1}{(2\ )}$

$x = \frac{7 \pm \sqrt{41}}{2} \qquad \left(\frac{7+\sqrt{41}}{2}\right)^3 + \frac{8}{\left(\frac{7+\sqrt{41}}{2}\right)^3} \longrightarrow$

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$h(t) = -10t^2 + 50t$

ارتفاع max توپ ← $y_{max} = \frac{-\Delta}{4a} = \frac{-(50)^2}{4 \times -10} = \frac{2500}{40} = \boxed{62.5\ m}$

زمانی که توپ به زمین برخورد می کند ← $h(t) = 0 \qquad -10t^2 + 50t = 0 \qquad -10t(t-5) = 0$

$t = 0$ × یا $\boxed{t = 5}$ ✓

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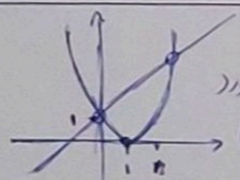
الف) $(x-1)^2 = 2x + 1$



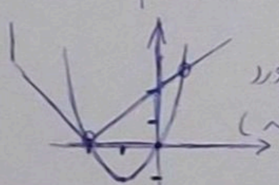
دو نقطه برخورد (دو ریشه)

$x^2 - 2x + 1 = 2x + 1$

$x^2 - 4x = 0 \qquad x(x-4) = 0$

$x = 0 \qquad x = 4$

ب) $x^2 + 2x = |x + 2|$



دو نقطه برخورد (دو ریشه)

$x = -2$

$x = 1$