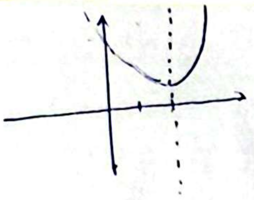
آیناز کریم‌زاده - یازدهم دختر B - تکلیف ۱۳

سوال ۱)



$\rightsquigarrow \text{ext}\left| -\frac{b}{2a} = 2 \to \frac{-1}{2a-2} = 2 \to 4a-4=-1 \right.$

$4a = 3 \to a = \frac{3}{4}$

$\to$ معادله سهمی $\Rightarrow -\frac{1}{4}x^2 + x + 3 = y$

$\to -\frac{1}{4}x^2 + x + 3 = 0 \to x^2 - 4x - 12 = 0 \begin{cases} \boxed{x = 6} \\ x = -2 \end{cases}$

در طول مثبت

سوال ۲)

x		x_1		x_2	
$f(x)$	$-$	0	$+$	0	$-$

$f(x) = mx^2 - 5x + m \to f($بین $x_1, x_2) = (+) \Rightarrow \boxed{m<0}$

$\frac{m}{m} = 1 > 0 \rightsquigarrow \alpha, \frac{1}{\alpha}$

سوال ۳)

$y = ax^2 + bx + c$

$\alpha = \frac{3+\sqrt{5}}{3} \quad \beta = \frac{3-\sqrt{5}}{3} \to \alpha + \beta = \frac{-b}{a} = 2 \to -b = 2a \to b = -2a$

$\alpha\beta = \frac{(3+\sqrt{5})(3-\sqrt{5})}{9} = \frac{9-5}{9} = \frac{4}{9} = \frac{c}{a} \Rightarrow 4a = 9c$

$\to y = x^2 - 2x + \frac{4}{9}$

$\to y = 9x^2 - 18x + 4$

سوال ۴)

$2x^2 - 8x + m + 2 = 0 \to 2\alpha^2 - 8\alpha + 4\alpha + 2 = 0 \to 2(\alpha-1)^2 = 0 \to \alpha = 1 \to 2 - 8 + m + 2 \to \boxed{m=4}$ ✓

$\alpha < \beta$

$3\alpha^2 + \beta^2 = 14 \to \alpha^2 + \beta^2 + 2\alpha^2 + 2 = 14 \to \underbrace{(\alpha+\beta)^2}_{16} - \underbrace{2\alpha\beta}_{-m-2} + 8\alpha - m = 14 \to 16 - m + 8\alpha - m = 14$

$4\alpha = m$

سوال ۵)

$y = ax^2 + bx + 5 \rightsquigarrow \text{ext}\left| \begin{matrix} 2 = \frac{-b}{2a} \to -b = 4a \\ 9 \end{matrix} \right.$

$-\frac{\Delta}{4a} = \frac{4ac - b^2}{4a} = 9 \to \frac{-5b - b^2}{-b} = 9 \to 9b = 5b + b^2 \to b^2 - 4b = 0 \begin{cases} b = 0 \ ✗ \\ b = 4 \ ✓ \end{cases} \to a = -1$

$y = -x^2 + 4b + 5 = -(x-5)(x+1) \to (-1, 5)$ در بازه

سوال ۶)

$\alpha x^2 - 7x + 9\beta = 0$

$\beta \rightsquigarrow (+)$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{7}{\alpha}}{\frac{9\beta}{\alpha}}$

$\to \alpha\beta = \frac{9\beta}{\alpha} \to \alpha^2 = 9 \begin{cases} \alpha = 3 \\ \alpha = -3 \end{cases}$

$\to \alpha + \beta = \frac{7}{\alpha} \to \underbrace{\alpha^2}_{9} + \alpha\beta = 7 \to \alpha\beta = -2 \Rightarrow \alpha = -3$

$\beta = \frac{2}{3}$

$\Rightarrow \frac{-\frac{7}{3}}{-\frac{9}{3}} \to \boxed{\frac{7}{9}}$ ✓

سوال ۷)

$x^2 + mx - 2m = 0 \xrightarrow{\beta} \beta^2 + m\beta - 2m = 0 \Rightarrow \beta^2 - 2m = -m\beta$

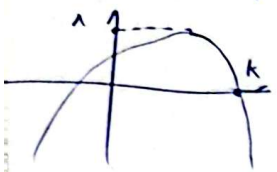
$\alpha^2 - m\beta = 8 \to \alpha^2 + \beta^2 - 2m = 8 \Rightarrow m^2 + 2m - 8 = 0 \to (m+4)(m-2) \begin{cases} m = -4 \to x^2 - 4x + 8 = 0 \quad \Delta < 0 \\ m = 2 \to x^2 + 2x - 4 = 0 \quad \Delta > 0 \ ✓ \end{cases}$

$\alpha + \beta$

$\alpha + \beta = \boxed{-2}$

سوال ۱۱

$f(x)=mx^2+4x+\frac{m}{2}+1$ $\rightarrow -\frac{\Delta}{4a}=-\frac{b^2-4ac}{4a}=\frac{-16+4m(\frac{m}{2}+1)}{4m}=8$

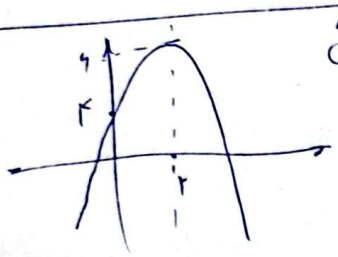


$\rightarrow -8+m^2+4m=16m \rightarrow m^2-12m-8=0 \begin{cases} m=-2 & a<0 \checkmark \\ m=6 & a>0 \times \end{cases}$

$f(x)=-2x^2+12x+9=-2(x^2-6x-9)$

$=-2(x-3)(x+1) \rightarrow \boxed{k=3}$

سوال ۱۲



$y=ax^2+bx+4 \rightarrow -\frac{1}{2}x^2+2x+4=0$

$-\frac{b}{2a}=2 \rightarrow -b=4a \xrightarrow{\times 4} 4a-8a+4=9 \rightarrow -4a=1$

$a=\frac{-1}{2} \rightarrow b=2$

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{4}{-8}=\frac{-1}{2}$

سوال ۱۳

$x^2-(4a+4)x+(12a^2+9a+3)=0 \rightarrow 4-8a-8+12a^2+9a+3=0 \rightarrow 12a^2-4a-1=0$

$\begin{cases} a=1 \rightarrow \alpha \rightarrow \text{ریشه منفی} \\ a=\frac{-1}{3} \rightarrow \checkmark \end{cases}$

$\alpha=2$ $\quad y=x^2-\frac{8}{3}x+\frac{4}{3}$

$\alpha\beta=\frac{4}{3}$ $\quad \alpha=2 \rightarrow \boxed{\beta=\frac{2}{3}}$