① $y=(a-1)x^2+x+3$ $\qquad x=2$

$2=-\frac{1}{2(a-1)} \Rightarrow 4(a-1)=-1 \rightarrow a-1=-\frac{1}{4} \rightarrow a=\frac{3}{4}$

$y=-\frac{1}{4}x^2+x+3 \longrightarrow x^2-4x-12=0$ [illegible]

$(x-6)(x+2)$

$x=6 \qquad x=-2$

$6-(-2)=8$

② $mx^2-5x+m \longrightarrow$

x		x_1		x_2	
		$-$	$+$	$-$	

$m<0$

③ $x_1=\frac{3+\sqrt{5}}{3} \qquad x_2=\frac{3-\sqrt{5}}{3}$

$x^2-\underbrace{(x_1+x_2)}_{S}x+\underbrace{x_1x_2}_{P}=0$

$x_1+x_2=\frac{6}{3}=2 \qquad x_1\times x_2=\frac{9-5}{9}=\frac{4}{9}$

$\xrightarrow{\times 9} 9x^2-18x+4=0$

④ $2x^2-8x+m+2=0 \qquad \alpha<\beta$

$3\alpha^2+\beta^2=12 \qquad m?$

$\begin{cases}\alpha+\beta=\frac{8}{2}=4\\ \alpha\beta=\frac{m+2}{2}\end{cases}$

$\alpha^2=(\alpha+\beta)^2-2\alpha\beta-\beta^2$

$\Rightarrow 3(\alpha+\beta)^2-6\alpha\beta-2\beta^2=12$

$\alpha\beta=3$

$m+2=6 \longrightarrow m=4$

$\alpha+\beta=4$

$3(16)-6\alpha\beta-2\beta^2=12$

$6\alpha\beta+2\beta^2=36$

$\alpha=4-\beta$

$3(4-\beta)\beta+\beta^2=18$

$-2\beta^2+12\beta-18=0$

$(\beta-3)^2=0 \Rightarrow \beta=3$

$\alpha=1$

(۳) $(۲, ۹)$ رئوس سهمی

$y = a(x-h)^۲ + k \rightarrow y = a(x-۲)^۲ + ۹$

$y = ۵ \rightarrow$ قطع $\rightarrow (۰, ۵)$ گذراز

$۵ = a(۰-۲)^۲ + ۹ \rightarrow ۵ = ۴a + ۹ \rightarrow ۴a = -۴ \rightarrow$ $a = -۱$

$y = -۱(x-۲)^۲ + ۹ \rightarrow -(x-۲)^۲ + ۹ > ۰$

$۹ > (x-۲)^۲ \rightarrow -۳ + ۲ < x < ۳ + ۲$

$-۱ < x < ۵$

$(-۱, ۵)$ ←

(۴) $\alpha, \beta \rightarrow ax^۲ - ۷x + ۹\beta = ۰ \rightarrow \beta$ ؟

$\frac{۱}{\alpha} + \frac{۱}{\beta} = ?$

$\alpha + \beta = \frac{۷}{a}$ $\quad \frac{۹}{a} + \beta = \frac{۷}{a}$

$\alpha\beta = \frac{۹\beta}{a}$ $\quad \beta = \frac{۷}{a} - \frac{۹}{a}$

$a = \frac{۹}{\alpha}$ $\quad \beta = -\frac{۲}{a}$

$\beta = -\frac{۲}{a} > ۰ \rightarrow a < ۰ \rightarrow \frac{۱}{\beta} + \frac{۱}{\alpha} = \frac{۱}{-\frac{۲}{a}} + \frac{۱}{\frac{۹}{a}}$

$= -\frac{a}{۲} + \frac{a}{۹} \rightarrow -\frac{۹a}{۱۸} + \frac{۲a}{۱۸}$

$-\frac{۷a}{۱۸}$ ←

تمرین

(۷) $x^2 + mx - 2m = 0$ $\quad \alpha + \beta = ?$ $\quad \alpha^2 - m\beta = 1$

$\alpha + \beta = -m \Rightarrow \alpha\beta = -2m$

$\alpha^2 + m\alpha - 2m = 0$

$\alpha^2 + m^2 + m\alpha = 1$

$\alpha^2 + m\alpha = 2m$

$m^2 + 2m - 1 = 0 \longrightarrow (m+3)(m-2) = 0 \longrightarrow \alpha + \beta = -m$

$m = 2 : \alpha + \beta = -2$ ✓

$m = -3 \longrightarrow \alpha + \beta = 3$ ✗

$\beta = -m - \alpha$

$\alpha^2 - m(-m-\alpha) = 1$

$\alpha(-m-\alpha) = -2m$

$\alpha^2 + \alpha m - 2m = 0$

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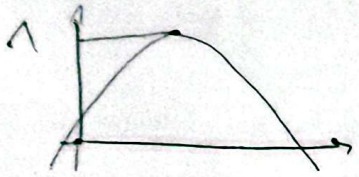
(۸) $f(x) = mx^2 + 3x + \frac{m}{2} + 1$ ✓



$\longrightarrow (K, 1)$

$x = -\frac{b}{2a} = -\frac{3}{2m} = -\frac{2}{m} = K$

$f\left(-\frac{2}{m}\right) = m\left(-\frac{2}{m}\right)^2 + 3\left(-\frac{2}{m}\right) + \frac{m}{2} + 1 = 1$

$m\left(\frac{4}{m^2}\right) - \frac{6}{m} + \frac{m}{2} + 1 = 1 \longrightarrow \left(-\frac{2}{m} + \frac{m}{2} - 1 = 0\right) \times 2m \longrightarrow -4 + m^2 - 2m = 0$

$(m-3)(m+2) = 0$

$m < 0 \longrightarrow \boxed{m = -2}$

$\boxed{K = -\frac{2}{-2} = 1}$

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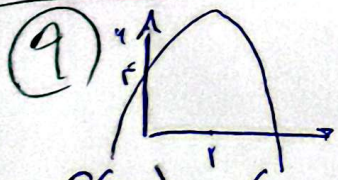
(۹) 

$a < 0 \longrightarrow S = (2, 4)$

$f(0) = 3$

$f(x) = a(x-2)^2 + 4$

$3 = a(0-2)^2 + 4 \longrightarrow 3 = 4a + 4 \longrightarrow 4a = -1 \longrightarrow \boxed{a = -\frac{1}{4}}$

$f(x) = -\frac{1}{4}(x-2)^2 \longrightarrow -\frac{1}{4}(x-2)^2 + 4 = 0 \longrightarrow (x-2)^2 = 16$

$x - 2 = \pm\sqrt{16} = \pm 4\sqrt{1} \qquad \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 x_2}$

$f(x) = ax^2 + bx + c \longrightarrow x_1 + x_2 = -\frac{b}{a} \qquad x_1 x_2 = \frac{c}{a}$

$x_1 + x_2 = 2 + 2\sqrt{3} + 2 - 2\sqrt{3} = 4$

$x_1 x_2 = (2 + 2\sqrt{3})(2 - 2\sqrt{3}) = 4 - 12 = -8$

$\frac{1}{x_1} + \frac{1}{x_2} = \frac{4}{-8} = -\frac{1}{2}$

(۱۰) $x^2 - (4a+4)x + (3a^2+6a+2) = 0$ بازاری

$x_1 = 2$ $\quad x_2 = ?$

$S = 4a+4$

$P = (3a^2+4a+2) \longrightarrow 2(2a+2) = 3a^2 + 6a + 3 \longrightarrow 3a^2 - 2a - 1 = 0$

$2 + x_1 = 4a+4 \Rightarrow x_2 = 4a+2$

$$a = \frac{2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)} = \frac{2 \pm \sqrt{4+12}}{6} = \frac{2 \pm 4}{6}$$

$x_2 = 4a+2$

$a = 1 \longrightarrow x_2 = 4(1) + 2 = 6$ ✓

$a = -\frac{1}{3} \Rightarrow x_2 = 4(-\frac{1}{3}) + 2 = -\frac{4}{3} + 2 = \frac{2}{3}$ ✓ x_2