

دقیق به نسبت به خط متعادل است یعنی آن خط از رأس می‌گذرد.

$$\text{رأس می: } \frac{-b}{2a} = 2 \Rightarrow \frac{-1}{2(a-1)} = 2 \rightarrow -1 = 4a - 4 \rightarrow 4a = 3 \rightarrow a = \frac{3}{4}$$

$$\rightarrow y = -\frac{1}{4}x^2 + x + 3 = 0 \rightarrow -x^2 + 4x + 12 = 0 \rightarrow x^2 - 4x - 12 = 0 = (x-6)(x+2) \rightarrow x = 4, -2$$

صورت مثبت $\rightarrow 4$

۱) معادله $mx^2 - 5x + m = 0$ دو ریشه دارد یعنی $\Delta > 0$

$$\Delta = (5)^2 - 4(m)(m) = 25 - 4m^2 \rightarrow 25 - 4m^2 > 0 \rightarrow 4m^2 < 25 \rightarrow m^2 < \frac{25}{4}$$

$$\Rightarrow -\frac{5}{2} < m < \frac{5}{2}$$

۲) با توجه به اینکه معادله در بازه‌ی بین دو معرناج مثبت است پس ریشه x متناهی است. $m < 0$

$$1 \wedge 2 \rightarrow \frac{0}{-\frac{5}{2} \quad 0 \quad \frac{5}{2}} \rightarrow -\frac{5}{2} < m < 0$$

$$S = \frac{3-\sqrt{5}}{2} + \frac{1+\sqrt{5}}{2} = \frac{1+2}{2} = 2$$

$$P = \left(\frac{3-\sqrt{5}}{2}\right)\left(\frac{1+\sqrt{5}}{2}\right) = \frac{(3-\sqrt{5})(1+\sqrt{5})}{4} = \frac{9-5}{4} = \frac{4}{4} = 1$$

$$Q = a(x^2 - Sx + P) = a(x^2 - 2x + 1) \rightarrow \text{چون فراب صبح است} \rightarrow u = 9$$

$$\Rightarrow 4x^2 - 12x + 4 = 0$$

$$3\alpha^2 + \beta^2 = 1\alpha^2 + \alpha^2 + 1\beta^2 - \beta^2 = 1\alpha - (m+2) + 1\beta - (m+2) + (\alpha^2 - \beta^2)$$

$$= 1(\alpha + \beta) + (\alpha - \beta)(\alpha + \beta) - 2(m+2)$$

$$\alpha + \beta = \frac{-(1-1)}{2} = 0, \quad \alpha - \beta = \frac{-\sqrt{\Delta}}{2a} = \frac{-\sqrt{\Delta}}{2} \rightarrow \Delta = (1)^2 - 4(m+2)(2) = 4 - 8m - 16$$

$$\rightarrow 1(0) + \left(\frac{-\sqrt{\Delta}}{2}\right)^2 - 2(m+2) = 12 \div 2, \quad 14 - \sqrt{\Delta} - (m+2) = 4$$

$$\Rightarrow \sqrt{\Delta} = 14 - 4 - m - 2 = 8 - m \rightarrow \Delta = (8-m)^2 \Rightarrow 4 - 8m - 16 = 64 - 16m - 14m \rightarrow m^2 - 16m + 14 = (m-4)^2 = 0 \Rightarrow m = 4$$

$$y = a(x-h)^2 + k \quad \text{رأس می: } (h, k) \rightarrow y = a(x-2)^2 + 9 = a(x^2 + 4 - 4x) + 9$$

$$= ax^2 + 4a - 4ax + 9 \quad \text{عوض از سر: } 0 = 4a + 9 \Rightarrow 4a = -9 \rightarrow a = -\frac{9}{4}$$

$$\rightarrow y = -\frac{9}{4}x^2 + 9x + 9$$

$$\rightarrow \frac{-1 \quad 9}{-9 \quad 0} \rightarrow \text{ریشه‌ی } (-1, 9) \text{ با یی گزیر ها}$$

$$x^2 - 4x - 9 = 0 = (x-5)(x+1)$$

$$\text{ریشه‌های باغ} \rightarrow 5, -1$$

$\alpha\beta = \frac{9\beta}{\alpha} \Rightarrow \alpha^2 = 9 \rightarrow \alpha = \pm 3 \rightarrow \begin{cases} \alpha = -3 \rightarrow S' = \frac{-9}{-3} = 3 = -3 + \beta \rightarrow \beta = \frac{-9}{-3} + 3 = \frac{0}{-3} \rightarrow \beta = 0 \checkmark \\ \alpha = 3 \rightarrow S' = \frac{9}{3} = 3 = 3 + \beta \rightarrow \beta = \frac{9}{3} - 3 = 0 \rightarrow \beta = 0 \checkmark \end{cases}$ $\Rightarrow \alpha = -3, \beta = \frac{1}{3} \rightarrow \frac{1}{-3} + \frac{1}{\frac{1}{3}} = \frac{-1}{3} + \frac{3}{1} = \frac{-1+9}{3} = \frac{8}{3}$ $\therefore \rho = \frac{9\beta}{\alpha} = \frac{9(\frac{1}{3})}{-3} = \frac{3}{-3} = -1$ $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-3+\frac{1}{3}}{-1} = \frac{-\frac{8}{3}}{-1} = \frac{8}{3}$	6
$x^r = rm - m\alpha \rightarrow \alpha^r - m\beta = rm - m\alpha - m\beta = rm - m(\alpha + \beta) = \lambda$ $\rightarrow S = \alpha + \beta = \frac{-m}{1} = -m \rightarrow rm - m(-m) = rm + m^2 = \lambda \rightarrow m^2 + rm - \lambda = 0$ $= (m+2)(m-2) = 0 \Rightarrow m = 2, -2 \quad \Delta > 0 \rightarrow \Delta = m^2 + 4\lambda \Rightarrow m(m+4) = \frac{-\lambda}{1+1+1} = \frac{-\lambda}{3}$ $\rightarrow m = 2, S = -m = -2$	7
$\frac{-\Delta}{\epsilon a} \rightarrow \text{مقسوم} \rightarrow \frac{-\Delta}{\epsilon a} = \lambda \rightarrow \Delta = 14 - \epsilon(\frac{m}{\epsilon} + 1)(m) = 14 - (rm + r\lambda)(m)$ $= 14 - rm^2 - r\lambda m \rightarrow \frac{-\Delta}{\epsilon a} = \frac{rm^2 + r\lambda m - 14}{\epsilon m} = \lambda$ $\rightarrow m^2 + 1\epsilon m - \lambda = f(\epsilon m) = 14m \rightarrow m^2 - rm - \lambda = (m-2)(m+2) = 0 \rightarrow m = 2, m = -2$ $\rightarrow -rx^r + \epsilon x + 4 = 0 \rightarrow rx^r - \epsilon x + 4 = 0 \quad \begin{cases} m = 2 \rightarrow rm^2 + \epsilon x + 4 \rightarrow \Delta < 0 \times \\ m = -2 \rightarrow -rm^2 + \epsilon x + 4 \rightarrow \Delta > 0 \checkmark \end{cases}$ $\rightarrow x^r - rx - 4 = (x-2)(x+2) = 0 \rightarrow x = \frac{2}{r} - 1$	8
$y = a(x-h)^r + k \quad (h, k) \rightarrow \text{مركز}$ $\rightarrow y = a(x-2)^r + 4 = a(x^r + \epsilon - \epsilon x) + 4 = ax^r + \epsilon a - \epsilon ax + 4$ $\rightarrow \text{مركز} = K = (\epsilon a + 4) \rightarrow \epsilon a = -2 \rightarrow a = \frac{-1}{\epsilon}$ $\rightarrow y = -\frac{1}{\epsilon}x^r + rx + \epsilon \rightarrow -\frac{1}{\epsilon}x^r + rx + \epsilon = 0$ $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} \rightarrow S' = \frac{-r}{-\frac{1}{\epsilon}} = \epsilon, \alpha\beta = \frac{\epsilon}{-\frac{1}{\epsilon}} = -\epsilon \rightarrow \frac{\epsilon}{-\epsilon} = -1$	9
$x = r \rightarrow (r)^r - (\epsilon a + \epsilon)x^r + (\epsilon a^r + 4a + r) = 0$ $\rightarrow r - \epsilon a - 1 + \epsilon a^r + 4a + r = \epsilon a^r - r\epsilon a - 1 = 0 \rightarrow 9a^r - 4a - 1 = (\epsilon a - 3)(\epsilon a + 1) = 0$ $\rightarrow a = 1, -\frac{1}{\epsilon}$ $a = 1 \rightarrow x^r - (\epsilon + \epsilon)x + (\epsilon + 4 + r) = x^r - \epsilon x + 12 = 0 = (x-4)(x-2) \Rightarrow x = 4$ $a = -\frac{1}{\epsilon} \rightarrow x^r - (\epsilon(-\frac{1}{\epsilon}) + \epsilon)x + (\epsilon(-\frac{1}{\epsilon})^r + 4(-\frac{1}{\epsilon}) + r) = 0 = x^r - \frac{\epsilon}{\epsilon}x + \frac{\epsilon}{\epsilon} = x^r - x + 1 = 0$ $\rightarrow 9x^r - r\epsilon x + 12 = 0 \Rightarrow (12x - 4)(12x - 2) = 0$ $\rightarrow x = \frac{2}{\epsilon} \rightarrow \text{مركز}$	10