

سایه جبری

$$f(x) = \begin{cases} x^3 - 4 & x > a \\ 12x - 20 & x \leq a \end{cases} \rightarrow x \leq a \rightarrow 12a - 20 \leq a^3 - 4$$

$$x \rightarrow f(x) = x^3 - 12x + 14 \leq 0 \rightarrow a^3 - 12a + 14 \leq (a-2)(a^2 + 2a - 1)$$

$$(a-2)^2 (a+2) \geq 0 \rightarrow a+2 \geq 0 \rightarrow \boxed{a \geq -2}$$

$$a^2 + 2a - 1 \leq (a-2)(a^2 + 2a - 1) \rightarrow a(a-2) \leq 0 \rightarrow \boxed{a \in [0, 2]}$$

$$f(x) = 3x + k \rightarrow f(\epsilon) = 3\epsilon + k \leq k \rightarrow \boxed{k \leq -1}$$

$$f(v) = 3(v) - 1 \leq 21 - 1 \leq 20$$

$$f(f(x)) \leq f(3x-1) \leq 3(3x-1)-1 \leq 9x-4 \leq 9x-4$$

$$f(x) = 3x-1$$

$$A(2a, 1) \quad f(x) = \frac{ax}{x-1} \rightarrow f(a) = \frac{a^2}{a-1}$$

$$f(a) = \frac{a^2}{a-1} \rightarrow a^2 \leq \frac{a^2}{a-1} \leq a^2 \leq \frac{a^2}{a-1} \rightarrow a \leq 0 \rightarrow a \leq 0$$

$$S = \{(2, 2), (4, 4), (6, 6), (8, 8)\} \quad \mathcal{G} = \{(2, 2), (4, 4), (6, 6), (8, 8)\}$$

$$f = f(f(x)) = \{(2, 2), (4, 4), (6, 6), (8, 8)\}$$

$$f = \{(2, 2), (4, 4), (6, 6), (8, 8)\}$$

$$f \circ g^{-1} = f(g^{-1}(x)) = \{(1, 2), (3, 4), (5, 1)\}$$

$$g^{-1} \circ f = \{(2, 1), (4, 3), (1, 5)\}$$

$$h = \{(1, 2), (3, 4), (5, 1)\} \quad g = \{(2, 1), (4, 3), (1, 5)\} \quad (4)$$

$$h \rightarrow h(x) = \{(1, \frac{1}{2}), (3, \frac{1}{4}), (5, \frac{1}{1})\}$$

$$f \circ g^{-1} = g^{-1} \circ \{(1, 2), (3, 4), (5, 1)\}$$

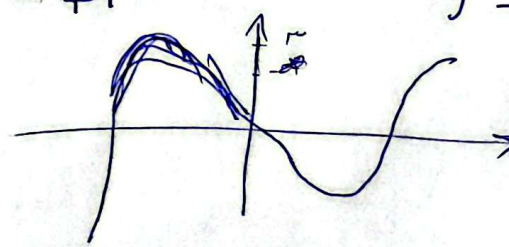
$$\hookrightarrow f(g^{-1}(x)) = \{(2, 1), (4, 3), (1, 5)\}$$

$$y = \frac{3x+1}{x-2} \rightarrow x = \frac{3y+1}{y-2} \rightarrow x(y-2) = 3y+1$$

$$xy - 2x = 3y + 1$$

$$xy - 3y = 2x + 1$$

$$y(x-3) = 2x+1$$

$$f^{-1}(x) = \frac{2x+1}{x-3} \leftarrow y = \frac{2x+1}{x-3}$$


$$f(x) = |x-1| - |x-3|$$

$$(1) x \leq 1 \rightarrow |x-1| = 1-x, |x-3| = 3-x$$

$$(2) 1 < x \leq 3 \rightarrow |x-1| = x-1, |x-3| = 3-x$$

$$f(x) = (x-1) - (3-x) = 2x-4$$

$$(3) |x-1| = x-1 \rightarrow |x-3| = x-3$$

$$f(x) = (x-1) - (x-3) = 2 \rightarrow x \in [1, 3]$$

$$f^{-1}(x) = \frac{x+1}{2} \rightarrow x \in [-5, 5]$$

$$f(x) = \begin{cases} x^3 + 1 & x > 1 \\ x-1 & x \leq 0 \end{cases} \rightarrow y = x^3 + 1 \rightarrow x = \sqrt[3]{y-1} \rightarrow f^{-1}(x) = \sqrt[3]{x-1} \rightarrow x > 1$$

$$y = x-1 \rightarrow x = \frac{y+1}{1} \rightarrow f^{-1}(x) = \frac{x+1}{1} \rightarrow x \leq -1$$

$$f^{-1}(x) = \begin{cases} \sqrt[3]{x-1} & x > 1 \\ \frac{x+1}{1} & x \leq -1 \end{cases}$$

$$f(x) = x^r - \frac{(x+1)^r}{x+r} \quad f^{-1}(x) = \frac{ax+b}{rx+d} \quad \text{سأجابه}$$

$$f^{-1}(b) = x \rightarrow f(x) = b$$

$$f\left(\frac{ax+b}{rx+d}\right) = x \rightarrow (x+1)^r = x^r + rx^{r-1} + rx + 1$$

$$f(x) = x^r - \frac{x^r + rx^{r-1} + rx + 1}{x+r} \rightarrow x(x+r) = x^r + rx^{r-1}$$

$$f(x) = x \left(\frac{x^r + rx^{r-1} - x^r - rx^{r-1} - rx - 1}{x+r} \right) = \frac{x^r + rx^{r-1} - rx - 1}{x+r}$$

$$f(x) = \frac{x}{x^r+1}$$

$$y = \frac{x}{x^r+1} \rightarrow x = \frac{y}{y^r+1}$$

$$x(y^r+1) = y \rightarrow xy^r - y + x = 0 \rightarrow y = \frac{1 \pm \sqrt{1-4x^r}}{2x}$$

$$f^{-1}(x) = \frac{1 \pm \sqrt{1-4x^r}}{2x}$$

$$f^{-1}(b) = \frac{x+1}{\frac{b(x+1)^r}{rx+d}} \quad (1)$$