Date : ۱۴۰۴ . ۷ . ۲۷

فاصله متقی

به نام خدا

۱) $x^3 - 4 = 12x - 20 \longrightarrow x^3 - 12x + 16 \longrightarrow (x^2 - 4x + 2)(x+4) \longrightarrow (a-2)^2(a+4) \geq 0$

$\hookrightarrow a \geq -4$

۲) $f(4) = 2 \quad f(x) = 3x + k \quad 2 = 12 + k \longrightarrow k = -10 \longrightarrow f(x) = 3x - 10$

الف) $f(7) = 3(7) - 10 = 11$

ب) $f(f(x)) = f(3x-10) = 3(3x-10) - 10 = 9x - 40$

۳) $f(x) = \frac{ax}{x-1} \longrightarrow 2a = \frac{a^2}{a-1} \Rightarrow a^2 = 2a^2 - 2a \rightarrow a^2 - 2a = 0$

$a(a-2) = 0 \longrightarrow a = 0$ غ ق ق ، $a = 2$

۴) الف) $f \circ f^{-1} \longrightarrow f(f^{-1}(x)) = \{(5,5), (7,7), (2,2), (4,4)\}$

ب) $f^{-1} \circ f \longrightarrow f^{-1}(f(x)) = \{(3,3), (4,4), (7,7), (9,9)\}$

ج) $f \circ g^{-1} \longrightarrow f(g^{-1}(x)) = \{(2,5), (5,4)\}$

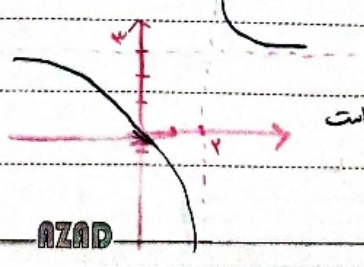
د) $g^{-1} \circ f \longrightarrow g^{-1}(f(x)) = \{(3,9), (7,3), (9,8)\}$

۵) $f \circ g^{-1} \longrightarrow f(g^{-1}(x)) = \{(1,4), (3,8), (5,0)\}$

$\frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{4}), (3, \frac{1}{8})\}$

۶) $y = \frac{3x+1}{x-2} \longrightarrow x = \frac{3y+1}{y-2} \Rightarrow 3y + 1 = xy - 2x \Rightarrow y(x-3) = 1 + 2x$

$y^{-1} = \frac{2x+1}{x-3}$



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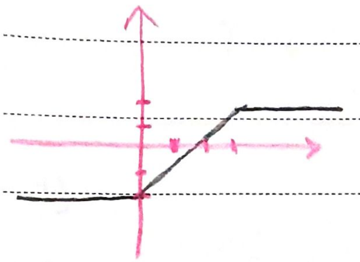
۷)

$f(x)=|x-1|-|x-3|$ آبیاری $[1,3]$ → $y=2x-4 \Rightarrow x=2y-4$

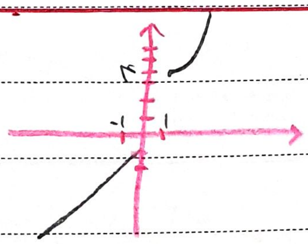
$x+4=2y \rightarrow y^{-1}=\dfrac{x+4}{2}$



→ $[a,b]=[1,3] \rightarrow \begin{matrix} a=1 \\ b=3 \end{matrix}$

۸)

$$f(x)=\begin{cases} x^3+4 & x\geq 1 \\ 4x-1 & x\leq 0 \end{cases}$$



⇐ وارون پذیر

$\xrightarrow{x\geq 1} y=x^3+4 \Rightarrow y\geq 5 \rightarrow x=y^3+4 \rightarrow y^3=x-4 \rightarrow y^{-1}=\sqrt[3]{x-4} \quad (x\geq 5)$

$\xrightarrow{x\leq 0} y=4x-1 \Rightarrow y\leq -1 \rightarrow x=4y-1 \rightarrow 4y=x+1 \rightarrow y^{-1}=\dfrac{x+1}{4} \quad (x\leq -1)$

$$f^{-1}(x)\begin{cases} \sqrt[3]{x-4} & x\geq 5 \\ \dfrac{x+1}{4} & x\leq -1 \end{cases}$$

۹)

$y=\dfrac{x^2(x+3)-(x+1)^3}{x+3}=\dfrac{x^3+3x^2-x^3-1-3x^2-3x}{x+3} \Rightarrow y=\dfrac{-3x-1}{x+3}$

$\hookrightarrow x=\dfrac{-3y-1}{y+3} \rightarrow xy+3x=-3y-1 \rightarrow y(x+3)=-3x-1 \Rightarrow y^{-1}=\dfrac{-3x-1}{x+3}\times\dfrac{2}{2}=\dfrac{-6x-2}{2x+6}$

$\hookrightarrow a=-6\,,\ b=-2\,,\ d=6 \Rightarrow f^{-1}(-2)=\dfrac{-6(-2)-2}{2(-2)+6}=\boxed{5}$

۱۰)

$y=\dfrac{x}{x^2+1} \rightarrow x=\dfrac{y}{y^2+1} \rightarrow y=xy^2+x \rightarrow xy^2-y+x=0 \rightarrow y=\dfrac{-b\pm\sqrt{\Delta}}{2a}$

$\hookrightarrow y^{-1}=\dfrac{1\pm\sqrt{1-4x^2}}{2x}$ ــ(x و y هم علامت / پس صورت باید +)→ $y^{-1}=\dfrac{1+\sqrt{1-4x^2}}{2x}\ ,\ x\in\left[\dfrac{-1}{2},\dfrac{1}{2}\right]$