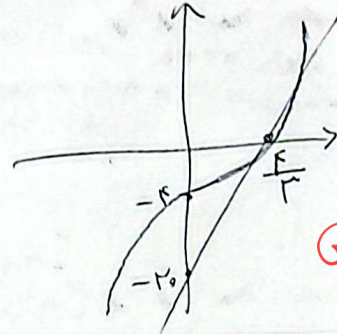


$$f(x) = \begin{cases} x^3 - 1 & x > 0 \\ 12x - 20 & x \leq 0 \end{cases}$$



$$a^3 - 1 > 12a - 20 \Rightarrow$$

$$(a-1)^2(a+2) > 0 \Rightarrow a+2 > 0 \Rightarrow a > -2$$

$$y = 3x + k \Rightarrow 12 + k = 2 \Rightarrow k = -10$$

$$3x - 10 = y \Rightarrow 3y - 10 = x$$

الف) $f(v) = (3 \times v) - 10 = 11$

ب) $f(f(x)) = f(3x - 10) = 3(3x - 10) - 10 = 9x - 40$

$$\frac{ax}{x-1} = y \Rightarrow ax = yx - y \Rightarrow y = x(y-a) \Rightarrow x = \frac{y}{y-a} \quad y \geq \frac{a}{a-1}$$

$$\frac{ya}{a} = a \Rightarrow a = 2$$

$$\frac{ya}{a} = a \Rightarrow ya = a^2 \Rightarrow a^2 - ya = 0 \Rightarrow a(a-y) = 0 \Rightarrow a = 0 \text{ (X)} \text{ or } a = y \text{ (check)}$$

$$f^{-1}(x) = \{(\omega, 1), (v, 4), (r, 9)\} \quad g^{-1}(x) = \{(2, 2), (4, 1), (\omega, 9)\}$$

الف) $f \circ f^{-1} = \{(\omega, \omega), (v, v), (r, r), (4, 4)\}$

ب) $f^{-1} \circ f = \{(2, 2), (v, v), (4, 4), (9, 9)\}$

ج) $f \circ g^{-1} = \{(2, \omega), (\omega, 9)\}$

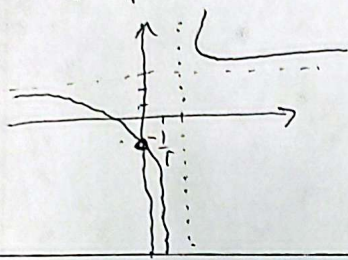
د) $g^{-1} \circ f = \{(r, 9), (v, 4), (9, 1)\}$

$$g^{-1} = \{(1, 2), (3, 4), (\omega, 4)\} \Rightarrow h = \{(1, 2), (3, 4), (4, 1), (\omega, 1)\}$$

$$f \circ g^{-1} = \{(1, 4), (3, 1), (\omega, 0)\} \quad \frac{h}{f \circ g^{-1}} = \frac{(1, 2)(3, 4)(\omega, 1)}{(1, 4)(3, 1)(\omega, 0)} \Rightarrow$$

$$\frac{h}{f \circ g^{-1}} = \frac{1}{\frac{1}{1} \cdot \frac{1}{3} \cdot \frac{1}{\omega}} = \frac{h}{f \circ g^{-1}} = \frac{1}{\frac{1}{3\omega}} \Rightarrow \{(1, \frac{1}{3\omega}), (3, \frac{1}{\omega})\}$$

$$y = \frac{2x+1}{x-2} \quad \text{هموار است}$$



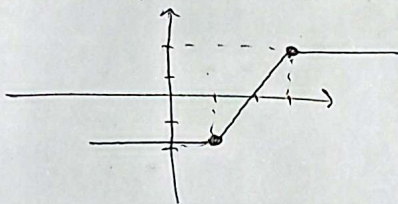
$$y = \frac{2x+1}{x-2} \Rightarrow yx - 2y = 2x+1 \Rightarrow x = \frac{-2y-1}{2y}$$

$$\text{معادله واردين} \quad y = \frac{2x+1}{x-2}$$

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(۵)

$$\begin{cases} x > 2 & x-1-x+2=1 \quad x=2 \\ 1 \leq x \leq 2 & x-1+x-2=2x-3=1 \quad x=2 \\ x < 1 & -x+1+x-2=-1 \quad x=2 \end{cases}$$



در بازه $[1, 2]$ معادله واردين است:

$$a=1 \quad b=2$$

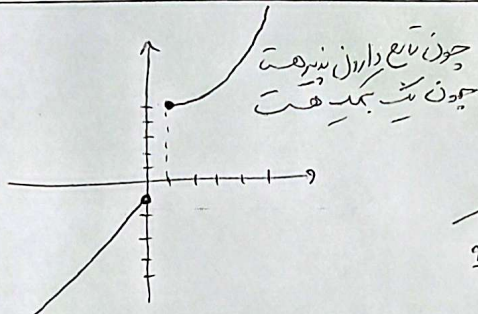
$$2x-3=y \Rightarrow 2x=y+3$$

$$x = \frac{y+3}{2} \Rightarrow y = \frac{2x-3}{1}$$

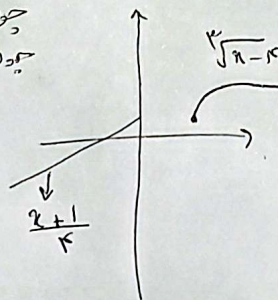
واردين

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(۵)



چون معادله واردين نيست
چون يك به يك است



$$x^2+1=y$$

$$x^2=y-1$$

$$x = \sqrt{y-1}$$

$$y = x^2+1$$

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$$2x-1=y$$

$$x = \frac{y+1}{2} = y = \frac{x+1}{2}$$

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$$\frac{x^2(x+1) - x^2 - 1 - 2x^2 - 2x}{x+1} = y \Rightarrow$$

$$\frac{x^3 + x^2 - x^2 - 1 - 2x^2 - 2x}{x+1} \Rightarrow$$

$$\frac{-x^2 - 2x - 1}{x+1} = y \Rightarrow y = \frac{-x^2 - 2x - 1}{x+1}$$

$$yx + 2y = -x^2 - 1$$

$$yx + 2x = -1 - 2y \quad x = \frac{-2y-1}{2y+1}$$

$$\frac{-2y-1}{2y+1} = \frac{ax+b}{cx+d} = 0$$

$$\frac{4x^2 - 4d^2 - 2x - 1}{2x+1} = \frac{ax+b}{cx+d} = 0 \quad f(b) = \frac{ab+b}{cb+d} = \frac{(-2-1)-1}{-2+1} = 0$$

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(۵)

$$P(x) = \frac{x}{x^2+1} \Rightarrow y(x^2+1) = x \Rightarrow yx^2 - x + y = 0 \xrightarrow{\Delta} \Delta = 1 - 4y^2$$

$$x = \frac{1 \pm \sqrt{1-4y^2}}{2y} \Rightarrow 1-4y^2 \geq 0 \Rightarrow -4y^2 \geq -1 \Rightarrow 4y^2 \leq 1 \Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$$

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(۵)

$$f(x)^{-1} = \frac{1 \pm \sqrt{1-4x^2}}{2x} \Rightarrow f(x)^{-1} = \frac{1 + \sqrt{1-4x^2}}{2x} \Rightarrow x \in \left[-\frac{1}{2}, \frac{1}{2}\right], x \neq 0$$

مستقيم قبل و بعد