

الفصل ١١

بازدهم و ختم

الدرج ١٥

$$x=a \Rightarrow a^3 - \varepsilon \geq 12a - 20 \Rightarrow a^3 - 12a + 14 \geq 0 \Rightarrow (a-2)^2(a+7) \geq 0 \Rightarrow a \geq -\varepsilon$$

$$f^{-1}(2) = \varepsilon \rightarrow f(\varepsilon) = 2 \Rightarrow 2 = 1\varepsilon + k \Rightarrow k = -10$$

$$\text{و) } f(v) = 21 - 10 = 11$$

$$\text{ب) } f(f(x)) = 3(3x - 10) - 10 = 9x - 30 - 10 = 9x - 40$$

$$y = \frac{ax}{x-1} \rightarrow x = \frac{ay}{y-1} \Rightarrow xy - x = ay \Rightarrow y(x-a) = x \Rightarrow y = \frac{x}{x-a}$$

$$\Rightarrow a = \frac{ya}{ya-a} \Rightarrow a = 21$$

$$g(x) = \{(3,2), (1,4), (9,5)\} \rightarrow g^{-1}(x) = \{(2,3), (4,1), (5,9)\}$$

$$f(x) = \{(3,5), (4,7), (7,2), (1,4)\} \rightarrow f^{-1}(x) = \{(5,3), (7,4), (2,7), (4,1)\}$$

$$\text{و) } f(f^{-1}(x)) = \{(5,5), (7,7), (2,2), (4,4)\}$$

$$\text{ب) } f^{-1}(f(x)) = \{(2,3), (4,4), (7,7), (1,1)\}$$

$$\text{ج) } f(g^{-1}(x)) = \{(2,5), (5,4)\}$$

$$\text{د) } g^{-1}(f(x)) = \{(3,4), (7,3), (4,1)\}$$

$$f(x) = \{(2,4), (4,1), (4,0)\} \quad h = \{(1,2), (3,4), (4,1), (5,1)\}$$

$$g(x) = \{(2,1), (4,3), (4,5)\} \rightarrow g^{-1}(x) = \{(1,2), (3,4), (5,4)\}$$

$$\frac{h}{f(g^{-1}(x))} = \{(1, \frac{1}{4}), (3, \frac{1}{4})\}$$

$$\hookrightarrow \{(3,1), (1,4), (5,0)\}$$

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$y = \frac{r^2x+1}{x-r} \rightarrow$ Asymptotes

Asymptotes $\frac{r^2y+1}{y-r} = x \Rightarrow r^2y+1 = xy-r^2 \Rightarrow y = \frac{r^2x+1}{x-r}$

$f(x) = \begin{cases} r & x \geq r \\ rx - \varepsilon & 1 \leq x \leq r \\ -r & x \leq 1 \end{cases}$

$\Rightarrow [a, b] = [1, r]$

Asymptotes $x = ry - \varepsilon \Rightarrow ry = x + \varepsilon \Rightarrow y = \frac{x + \varepsilon}{r}$

$f(x) = \begin{cases} x^r + \varepsilon & x \geq 1 \\ \varepsilon x - 1 & x \leq 0 \end{cases}$

$\Rightarrow f(x)$ is continuous

① $y = x^r + \varepsilon \rightarrow x \geq 1 \quad R = [a, +\infty)$
 $x = y^{\frac{1}{r}} + \varepsilon \rightarrow y \geq a$
 $y = \sqrt[r]{x - \varepsilon}, y \geq a$

② $y = \varepsilon x - 1 \quad x \leq 0 \rightarrow R = (-\infty, b]$
 $x = \frac{y + 1}{\varepsilon}, y \leq -1$

$\therefore f(x) = \begin{cases} \sqrt[r]{x - \varepsilon}, & x \geq a \\ \frac{x + 1}{\varepsilon}, & x \leq -1 \end{cases}$

$f(x) = \frac{x^r - (x+1)^r}{x+r} \rightarrow \frac{x^r + rx^{r-1} - x^r - 1 - rx^{r-1} - r^2x^{r-2} - \dots - r^2x - r^2}{x+r} = \frac{-r^2x - 1}{x+r} \rightarrow x = \frac{-ry - 1}{y+r}$

$\rightarrow -ry - 1 = xy + rx \Rightarrow y(-r - x) = rx + 1 \rightarrow y = \frac{rx + 1}{-x - r} \xrightarrow{x \rightarrow -r} y = \frac{-4r - 1}{-r + r} = \frac{-4r - 1}{0}$

$a = -4, b = -5, d = 4$

$f'(-r) = \frac{4-1}{-r+r} = \frac{3}{0} = \Delta$

$y = \frac{x}{x^2+1} \rightarrow y(x^2+1) = x \rightarrow yx^2 + y - x = 0$

Domain $D_{f^{-1}} = [-\frac{1}{4}, \frac{1}{4}]$, $R_{f^{-1}} = [-1, 1]$, $D_f = [-1, 1]$, $R_f = [-\frac{1}{4}, \frac{1}{4}]$

$yx^2 + y - x = 0 \rightarrow x = \frac{1 \pm \sqrt{1 - 4y^2}}{2y} \quad y \neq 0, \quad y = \frac{1 \pm \sqrt{1 - 4x^2}}{2x} \quad x \neq 0$

$f^{-1}(x) = \frac{1 + \sqrt{1 - 4x^2}}{2x}, x \in [-\frac{1}{4}, \frac{1}{4}]$

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