

۱) $f(x) = \begin{cases} x^2 - 4 & ; x > a \\ 12x - 2 & ; x \leq a \end{cases}$ $R_{(1)} = (a^2 - 4, +\infty)$ $R_2 = (-\infty, 12a - 2]$ (۱)

$a^2 - 4 \geq 12a - 2 \Rightarrow a^2 - 12a + 14 \geq 0 \Rightarrow (a-2)(a+4) \geq 0$ (۲)

$\Rightarrow \frac{(a-2)^2(a+4)}{(a-2)(a+4)} \geq 0 \Rightarrow \boxed{a \geq -4}$ (۳)

$f(x) = 2x + k, f^{-1}(r) = 4$ (۴)

الف) $f(v) = ?$ $f(4) = r$ $f(4) = 2(4) + k = 12 + k \Rightarrow r = 12 + k \Rightarrow k = -1$ (۵)

$f(v) \Rightarrow f(v) = 2(v) - 1 = 2v - 1 = 11 \Rightarrow v = 6$

ب) $f(f(x)) = ?$ $\frac{2(2x-1)-1}{2x-4} = 4x-4 \Rightarrow$ (۶)

$A(1a, a)$ $f(x) = \frac{ax}{x-1}$ $a = ?$ $f(x) = \frac{a \cdot a}{a-1} = \frac{a^2}{a-1}$ $f(a) = ra$ (۷)

$ra = \frac{a^2}{a-1} \Rightarrow ra^2 - ra = a^2 \Rightarrow ra^2 - a^2 - ra = 0 \Rightarrow a^2 - ra = 0 \Rightarrow a(a-r) = 0 \Rightarrow a = 0$ (۸)

$a = 0$ یا $a = r$ $\Rightarrow a = r$ (۹)

$f = \{(3, 5), (4, 2), (5, 1), (6, 4)\}$ $f^{-1} = \{(5, 3), (2, 4), (1, 5), (4, 6)\}$ (۱۰)

$g = \{(3, 2), (4, 1), (5, 4)\}$ $f^{-1}(5) = 3 \rightarrow f(3) = 5 / f^{-1}(2) = 4 \rightarrow f(4) = 2$

الف) $f \circ f^{-1} = \{(4, 4), (5, 5), (6, 6), (3, 3)\}$ $f^{-1}(r) = v \rightarrow f(v) = r / f^{-1}(4) = 6 \rightarrow f(6) = 4$

ب) $f^{-1} \circ f = \{(3, 3), (4, 4), (5, 5), (6, 6)\}$ $f(4) = 2 \rightarrow f^{-1}(2) = 4 / f(5) = 1 \rightarrow f^{-1}(1) = 5 / f(6) = 4 \rightarrow f^{-1}(4) = 6$

ج) $f \circ g^{-1}$ $g^{-1} = \{(2, 3), (1, 4), (4, 5)\}$ $g^{-1}(5) = 4 \rightarrow f(4) = 2$ (۱۱)

$g^{-1}(2) = 3 \rightarrow f(3) = 5 / g^{-1}(4) = 1 \rightarrow f(1) = 5$ (۱۲)

د) $g^{-1} \circ f$ $f(3) = 5 \rightarrow g^{-1}(5) = 4 / f(4) = 2 \rightarrow g^{-1}(2) = 3 / f(5) = 1 \rightarrow g^{-1}(1) = 4$ (۱۳)

$f(6) = 4 \rightarrow g^{-1}(4) = 1$ (۱۴)

$f = \{(2, 4), (4, 1), (5, 0)\}$ $x = 2 \rightarrow g(2) = 1 \rightarrow f(1) = 5$ (۱۵)

$g = \{(2, 1), (4, 2), (5, 4)\}$ $x = 4 \rightarrow g(4) = 2 \rightarrow f(2) = 4$ (۱۶)

$h = \{(1, 2), (3, 4), (4, 1), (5, 0)\}$ $x = 5 \rightarrow g(5) = 4 \rightarrow f(4) = 2$ (۱۷)

$\frac{h}{f \circ g^{-1}} = ?$ (۱۸)

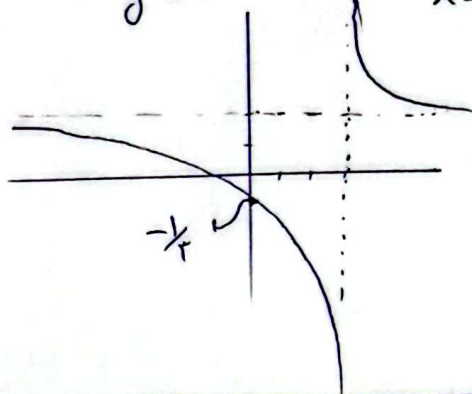
$$y = \frac{3x+1}{x-2} \quad \text{خط عمودی: } x=2$$

$$\text{خط افقی: } y=3$$

(4) تابع وارثانه است. یک به یک و معکوس پذیر

$$yx - 2y = 3x + 1 \Rightarrow yx - 3x = 1 + 2y \Rightarrow x(y-3) = 1+2y$$

$$\Rightarrow x = \frac{1+2y}{y-3} \Rightarrow y = \frac{1+3x}{x-2} \Rightarrow f(x) = \frac{1+3x}{x-2}$$



$$\begin{cases} x=2 \\ y=3 \end{cases}$$

$$f(x) = |x-1| - |x-3| \quad [a, b] \rightarrow \text{طول بازه}$$

$$f(x) = \begin{cases} x & x < 1 \\ -2x+4 & 1 \leq x \leq 3 \\ -2 & x > 3 \end{cases} \rightarrow \text{در بازه بازه، یک به یک و معکوس پذیر}$$

$$f(x) = \begin{cases} x^2+4 & x \geq 1 \\ 4x-1 & x \leq 1 \end{cases} \rightarrow \text{معکوس}$$

$$D_1 = [1, +\infty)$$

$$f(1) = 1^2 + 4 = 5$$

$$R_1 = [5, +\infty)$$

(5) بخش I معکوس پذیر

$$y = x^2 + 4 \rightarrow x = \sqrt{y-4}$$

$$f^{-1}(y) = \sqrt{y-4}$$

$$f^{-1}(x) = \begin{cases} \sqrt{x^2-4} & x \geq 5 \\ \frac{x+1}{2} & x \leq 1 \end{cases}$$

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$$y = 4x-1 \rightarrow x = \frac{y+1}{4} \rightarrow f^{-1}(y) = \frac{y+1}{4} \quad \text{بخش II}$$

$$f(x) = x^2 - \frac{(x+1)^2}{x+2}$$

$$f^{-1}(x) = \frac{ax+b}{cx+d} \quad f^{-1}(b)$$

$$\frac{x^2(x+2)}{x+2} - \frac{(x+1)^2}{x+2} = \frac{x^3 + 2x^2 - x^2 - 2x - 1}{x+2} = \frac{-x^2 - 2x - 1}{x+2}$$

$$y = \frac{-x^2-1}{x+2} \Rightarrow yx + 2y = -x^2 - 1 \Rightarrow yx + x^2 = -2y - 1 \Rightarrow x(y+2) = -2y-1 \Rightarrow$$

$$x = \frac{-2y-1}{y+2} \Rightarrow y = \frac{-2x-1}{x+2} \quad \text{از } y = \frac{-4x-2}{2x+4} \quad a=-4, b=-2, c=2$$

$$f^{-1}(b) = f^{-1}(-2) \Rightarrow f^{-1}(-2) =$$

$$\frac{-2(-2)-1}{-2+2} = \infty$$

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١٨ محرم ١٤٣٩

$$f(x) = \frac{x}{x^2 + 1}$$

$$Df = \mathbb{R}$$

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$$x = f^{-1}(y) \quad y = \frac{x}{x^2 + 1} \rightarrow yx^2 + y = x \Rightarrow$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(y)(y)}}{2y} = \frac{1 \pm \sqrt{1 - 4y^2}}{2y}$$

$$y = \frac{1 \pm \sqrt{1 - 4x^2}}{2x} \Rightarrow f^{-1}(x) = \frac{1 \pm \sqrt{1 - 4x^2}}{2x}$$