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$$f(x) = \begin{cases} x^2 - 4 & ; x > a \\ 12x - 2 & ; x \leq a \end{cases}$$

$$R_1 = (a^2 - 4, +\infty)$$

$$R_2 = (-\infty, 12a - 2]$$

$$a^2 - 4 \geq 12a - 2 \Rightarrow a^2 - 12a + 14 \geq 0 \Rightarrow (a - 2)(a + 10) \geq 0$$

$$\Rightarrow \frac{a(a-2)}{a-2} (a+4) \geq 0 \Rightarrow \boxed{a \geq -4}$$

$$f(x) = 2x + k, f^{-1}(r) = 4$$

$$\text{الف) } f(v) = ? \quad f(4) = r \quad f(4) = 2(4) + k = 12 + k \Rightarrow r = 12 + k \Rightarrow k = -1.$$

$$f(x) = 2x - 1$$

$$f(v) \Rightarrow f(v) = 2(v) - 1 = r - 1 = 11 \rightarrow \text{ب}$$

$$\text{ب) } f(f(x)) = ? \quad \frac{2(2x-1)-1}{2x-4} = 4x-4 \rightarrow \text{ج}$$

$$A(1a, a) \quad f(x) = \frac{ax}{x-1} \quad a = ? \quad f(x) = \frac{a \cdot a}{a-1} = \frac{a^2}{a-1} \quad f(a) = ra \quad \text{د}$$

$$ra = \frac{a^2}{a-1} \Rightarrow ra^2 - ra = a^2 \Rightarrow ra^2 - a^2 - ra = 0 \Rightarrow a^2 - ra = 0 \Rightarrow a(a-r) = 0 \Rightarrow a = 0 \text{ یا } a = r$$

$a = 0$ یا $a = r$ (چون $f(x) = 0$ باید در $x=1$ باشد)

$$f = \{(3,5), (4,6), (5,7), (6,8)\} \quad f^{-1} = \{(5,3), (6,4), (7,5), (8,6)\}$$

$$g = \{(3,2), (4,3), (5,4)\} \quad f^{-1}(5) = 3 \rightarrow f(3) = 5 / f^{-1}(6) = 4 \rightarrow f(4) = 6$$

$$\text{الف) } f \circ f^{-1} = \{(4,4), (5,5), (6,6), (7,7)\} \quad f^{-1}(7) = 5 \rightarrow f(5) = 7 / f^{-1}(8) = 6 \rightarrow f(6) = 8$$

$$\text{ب) } f^{-1} \circ f = \{(3,3), (4,4), (5,5), (6,6)\} \quad f(4) = 6 \rightarrow f^{-1}(6) = 4 / f(5) = 7 \rightarrow f^{-1}(7) = 5 / f(6) = 8 \rightarrow f^{-1}(8) = 6$$

$$\text{ج) } f \circ g^{-1} \quad g^{-1} = \{(2,3), (3,4), (4,5)\} \quad g^{-1}(5) = 4 \rightarrow f(4) = 6$$

$$g^{-1}(3) = 2 \rightarrow f(2) = 5 / g^{-1}(4) = 3 \rightarrow f(3) = 6$$

$$f \circ g^{-1} = \{(2,5), (3,6), (4,7)\}$$

$$\text{د) } g^{-1} \circ f \quad f(3) = 5 \rightarrow g^{-1}(5) = 4 / f(4) = 6 \rightarrow g^{-1}(6) = 3 / f(5) = 7 \rightarrow g^{-1}(7) = 4$$

$$f(6) = 8 \rightarrow g^{-1}(8) = 5 \quad (g^{-1} \circ f = \{(3,4), (4,5), (5,6)\})$$

$$f = \{(2,3), (3,4), (4,5)\}$$

$$x = 2 \rightarrow g(2) = 1 \rightarrow f(1) = 3$$

$$g = \{(2,1), (3,2), (4,3)\}$$

$$x = 3 \rightarrow g(3) = 2 \rightarrow f(2) = 3$$

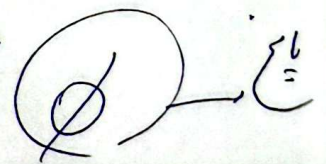
$$h = \{(1,2), (2,3), (3,4), (4,5)\}$$

$$x = 4 \rightarrow g(4) = 3 \rightarrow f(3) = 4$$

$$h \circ g^{-1} = ?$$

خرج تعریف شده در مورد ϕ تصویر می‌باشد

$$\{(1,1), (2,2), (3,3), (4,4)\}$$



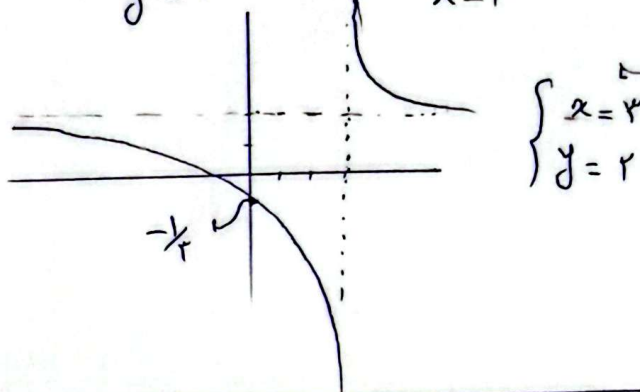
$$y = \frac{3x+1}{x-2} \quad \text{کتاب : } x=2$$

$$\text{کتاب : } y=3$$

(4) تابع معکوس پیدا کنید و نمودش بنویسید

$$yx - 2y = 3x + 1 \Rightarrow yx - 3x = 1 + 2y \Rightarrow x(y-2) = 1+2y$$

$$\Rightarrow x = \frac{1+2y}{y-2} \Rightarrow y = \frac{1+2x}{x-2} \Rightarrow f(x) = \frac{1+2x}{x-2}$$



(5) تابع در بازه $[1, 3]$ معکوس پذیر است. نمودش بنویسید.

$$f(x) = |x-1| - |x-3|$$

$$f(x) = \begin{cases} x & x < 1 \\ -2x+4 & 1 \leq x \leq 3 \\ -2 & x > 3 \end{cases}$$

در بازه $[1, 3]$ تابع نزاع و معکوس پذیر است.

$$f^{-1}(y) = \frac{y+4}{2} \quad -2 \leq y \leq 2$$

(6) نمودش I نمودش بنویسید

$$f(x) = \begin{cases} x^2+4 & x \geq 1 \\ 4x-1 & x \leq 0 \end{cases}$$

$$D_1 = [1, +\infty)$$

$$f(1) = 1^2+4=5$$

$$R_1 = [5, +\infty)$$

$$y = x^2+4 \rightarrow x = \sqrt{y-4}$$

$$f^{-1}(y) = \sqrt{y-4}$$

نمودش II

$$y = 4x-1 \rightarrow x = \frac{y+1}{4} \rightarrow f^{-1}(y) = \frac{y+1}{4}$$

(7) (5)

$$f(x) = x^2 - \frac{(x+1)^2}{x+2}$$

$$f^{-1}(x) = \frac{ax+b}{cx+d} \quad f^{-1}(b)$$

$$\frac{x^2(x+2)}{x+2} - \frac{(x+1)^2}{x+2} = \frac{x^3 + 2x^2 - x^2 - 2x - 1}{x+2} = \frac{-x^2 - 2x - 1}{x+2}$$

$$y = \frac{-x^2-2x-1}{x+2} \Rightarrow yx + 2y = -x^2 - 2x - 1 \Rightarrow yx + 2x = -y^2 - 1 \Rightarrow x(y+2) = -y^2 - 1$$

$$x = \frac{-y^2-1}{y+2} \Rightarrow y = \frac{-x^2-1}{x+2} \quad \frac{x^2}{x+2} \Rightarrow y = \frac{-x^2-1}{x+2}$$

$$a=-1, b=-1, c=1$$

$$f^{-1}(b) = f^{-1}(-1) \Rightarrow f^{-1}(-1) = \frac{-1^2-1}{-1+2} = \frac{-2}{1} = -2$$

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$$f(x) = \frac{x}{x^2 + 1}$$

$$Df = \mathbb{R}$$

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$$x = f^{-1}(y) \quad y = \frac{x}{x^2 + 1} \rightarrow yx^2 + y = x \Rightarrow$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(y)(y)}}{2y} = \frac{1 \pm \sqrt{1 - 4y^2}}{2y}$$

$$y = \frac{1 \pm \sqrt{1 - 4x^2}}{2x} \Rightarrow f^{-1}(x) = \frac{1 \pm \sqrt{1 - 4x^2}}{2x}$$