نام و نام خانوادگی [illegible] پاسخنامه تشریحی تکلیف شماره ۸ کلاس یازدهم ریاضی [illegible]

۱۹/۵

۱ (۱/۵)

$f(x) = \begin{cases} x^3 - 4 & x \geq a \\ 12x - 20 & x < a \end{cases}$

$x \geq a \Rightarrow f(a) = a^3 + 4$

$x < a \Rightarrow f(a) = 20 - 12a$

$-a^3 + 4 = 20 - 12a$

$-a^3 + 12a + 16 = 0$

$-a^3 + 12a + 16 \;\big|\; a + 2$

$a^2 - 2a + 8$

$a = 2$ ⟵ [illegible] , $a = -4$

$-x^3 + 4 \geq 12a + 20 \quad a \geq 2$ ✓

$a < 2$

$-4 \quad 24$

$- \quad + \quad + \quad +$

$a \in [-4, +\infty)$

۲ (۲)

$f(x) = 3x + k$

$f^{-1}(2) = 4 \rightarrow \text{if } f(\alpha) = \beta \Rightarrow f^{-1}(\beta) = \alpha$

$f(4) = 2 \quad f(4) = 12 + k = 2 \quad k = -10 \qquad f(x) = 3x - 10$

الف) $f(7) = 21 - 10 = 11$

ب) $f(3x - 10) = 3(3x - 10) - 10 = 9x - 40$

۳ (۳)

$f(x) = \dfrac{ax}{x-1}$

$f^{-1}(x) = \dfrac{x}{x-a}$

$f(2a) = \dfrac{2a}{2a - a} = a \Rightarrow a = 2$

$a^2 - 2a = 0 \quad a = 0$ غ ق ق

۴ (۲)

$g(x): \{(3,2)(8,6)(9,5)\} \quad g^{-1}(x): \{(2,3)(6,8)(5,9)\} \quad f(x): \{(2,5)(4,7)(7,3)(9,6)\}$

$f^{-1}(x): \{(5,2)(7,4),(3,7)(6,9)\}$

الف) $f(f^{-1}(x)) = \{(5,5)(7,7)(3,3)(6,6)\}$

ب) $f^{-1}(f(x)) = \{(2,2)(4,4)(7,7)(9,9)\}$

ج) $f(g^{-1}(x)) = \{(3,5)(5,6)\}$

د) $g^{-1}(f(x)) = \{(2,9)(7,3)(9,8)\}$

۵ (۳)

$f = \{(2,3)(4,8)(6,0)\}$

$g = \{(2,1)(4,3)(6,5)\}$

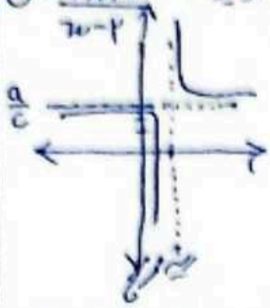
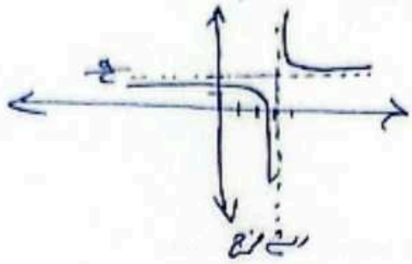
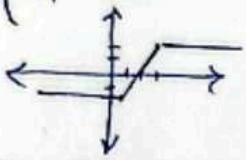
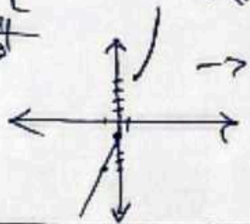
$g^{-1} = \{(1,2)(3,4)(5,6)\}$

$h = \{(1,2)(3,4)(4,1)(5,1)\}$

$fog^{-1} = \{(1,3)(3,8)(5,0)\}$

$\dfrac{h}{fog^{-1}} = \{(1, \tfrac{2}{3})(3, \tfrac{4}{8})(\cancel{5, \tfrac{1}{0}})\}$

$\dfrac{h}{fog^{-1}} = \{(1, \tfrac{2}{3}),(3, \tfrac{1}{2})\}$

$y=\frac{3x+1}{x-2}$ معکوس‌پذیر است $\quad f^{-1}: \frac{2y+1}{y-2}=x \quad y=-\frac{-2x-1}{x-3}=\frac{2x+1}{x-3}$  	(۲)	۶
$f=\|x-1\|-\|x-3\|$ $f(x)=\begin{cases} 2 & x\geq 3 \\ 2x-4 & 1\leq x<3 \\ -2 & x<1 \end{cases} \rightarrow y=2x-4$ $x=2y-4$ $y=\frac{x+4}{2} \quad R:[1,3] \quad D:[-2,2]$ 	(۲)	۷
$f(x)=\begin{cases} x^3+4 & x\geq 1 \\ 4x-1 & x\leq 0 \end{cases}$ $\rightarrow y=x^3+4 \quad x=y^3+4 \quad y=\sqrt[3]{x-4}$ $\rightarrow y=4x-1 \quad x=4y-1 \quad y=\frac{x+1}{4}$ $f^{-1}=\begin{cases} \sqrt[3]{x-4} & x\geq 5 \\ \frac{x+1}{4} & x\leq -1 \end{cases}$ وارون‌پذیر است 	(۲)	۸
$f(x)=x^2-\frac{(x+1)^3}{x+3}=\frac{x^3+3x^2-x^3-1-3x^2-3x}{x+3}=\frac{-3x-1}{x+3} \quad f^{-1}=\frac{-3x-1}{x+3}$ $b=-2 \quad f^{-1}(-2)=\frac{6-1}{-2+3}=5$	(۲)	۹
$f(x)=\frac{x}{x^2+1}$ $x=\frac{y}{y^2+1} \xrightarrow{} xy^2-y+x=0$ $y=\frac{+1\pm\sqrt{1-4x^2}}{2x}$ $1-4x^2\geq 0 \quad x\in[-\frac{1}{2},\frac{1}{2}]$	(۲)	۱۰