

$$(Roll) = n - n \leq r \rightarrow n - n + r \leq$$

$$\begin{array}{cc} (r, r) & (n-r)(n+1) \\ \downarrow & \downarrow \\ \boxed{msr} & \boxed{nsr} \quad ns-1 \times \end{array}$$

$$(m, r) \rightarrow (a, 1) \quad (-1+r, n)$$

$$(r) f(x)_s \begin{cases} rx-1 : x \geq 1 \rightarrow a+1 \\ x+a : x < 1 \rightarrow a+r \end{cases} \quad a \geq 0$$

$$x_{s1} \rightarrow F(1^-)_{s1} + a \rightarrow 1 + a < r \rightarrow \boxed{a < 1}$$

(iv) $f(x) = \begin{cases} x-1 & x \geq 1 \rightarrow a_s r \\ ax+a-1 & x \leq 0 \rightarrow a_s a \end{cases}$

$$f(x) \sim x^{a-1}$$

$$f(1) \leq \dots \leq f(r) \rightarrow [r, +\infty) \quad f(1) \leq f(1) \leq f(r) \quad (0 < a < r)$$

$$\text{Obj } \mathcal{C} \rightarrow F_0(0) \hookrightarrow F_0(1) \rightarrow a_1 \hookrightarrow a \hookrightarrow a_0$$

$$\searrow F(\cdot) \hookrightarrow F(1) \rightarrow a_1 \hookrightarrow a \hookrightarrow a_0$$

$$(*) \quad f(x) \in x^{p+1} \Rightarrow x_1 + 1, x_r + 1 \rightarrow x_1, x_r$$

$$y \circ \pi^w + r \circ \pi^w = y - r \Rightarrow x \circ \sqrt[y-r]{} \rightarrow f^{-1}(x) \circ \sqrt[x-r]{}$$

ب) $x' - f(x) \rightarrow D^s [2, +\infty)$

$$\Rightarrow x = 1 + \sqrt{y+1} \quad f^{-1}(x) = 1 + \sqrt{x-1}$$

5)

$$y \frac{3x+1}{x-2}$$

$$x-2 \rightarrow x \neq 2$$

یک به یک است

x	0	1	3	4
$\frac{1}{x-2}$	$-\frac{1}{2}$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{2}$
$\frac{3x+1}{x-2}$	$-\frac{5}{2}$	-4	4	5

$$x(y-2) \frac{3x+1}{x-2} \rightarrow x(y-2) \frac{3x+1}{x-2}$$

$$y(x-2) \frac{3x+1}{x-2}$$

$$y \frac{3x+1}{x-2} \rightarrow f^{-1}(x) \frac{3x+1}{x-2}$$

$$y \frac{3x+1}{x-2} \rightarrow x \neq 2$$

x	-1	0	1
$\frac{3x+1}{x-2}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{1}$

$$y \frac{3(x+2)}{x+2} = 3$$

تابع ثابت است $x \neq -2$

4) الف) $y \sqrt{x-3} \rightarrow x-3 \geq 0 \Rightarrow x \geq 3$ تابع یک به یک ✓

$$x \sqrt{y-3} \rightarrow x^2(y-3) \rightarrow y = x^2 + 3 \quad f^{-1}(x) = x^2 + 3 \quad x \geq 0$$

$$y = x^2 - 5x + 3 \quad x \in [2, +\infty)$$

یک به یک ✓

x	2	3
$x^2 - 5x + 3$	-1	-6

$$y = x^2 - 5x + 3 \rightarrow (x-2)^2 - 1$$

$$x = (y-1)^2 - 1 \rightarrow x+1 = (y-2)^2 \rightarrow y = 2 \pm \sqrt{x+1}$$

$$f^{-1}(x) = 2 + \sqrt{x+1} \quad x \geq -1$$

$$\textcircled{V} \quad x + \sqrt{x} \quad x \in (0, +\infty)$$

$$f(x) = ax + b - \sqrt{x - c}$$

$$f^{-1}(x) \Rightarrow x = y + \sqrt{y}$$

$$\sqrt{y} = t \Rightarrow y = t^2 \rightarrow x = t^2 + t$$

$$t^2 + t - x = 0 \quad t = \frac{-1 \pm \sqrt{1+4x}}{2}$$

$$t = \sqrt{y} \geq 0 \rightarrow t = \frac{-1 + \sqrt{1+4x}}{2}$$

$$y = t^2 \rightarrow y = \frac{1 - 2\sqrt{1+4x} + (1+4x)}{4} \rightarrow y = x + \frac{1}{4} - \frac{1}{4}\sqrt{1+4x}$$

$$f^{-1}(x) = a^2 + b\sqrt{x-c} \rightarrow x + \frac{1}{4} - \frac{1}{4}\sqrt{1+4x}$$

$$\frac{1}{4}\sqrt{1+4x} = \sqrt{x-c} \rightarrow \left(\frac{1}{4}\right)^2(1+4x) = x-c$$

$$a = 1 \quad b = \frac{1}{4} \quad x + \frac{1}{4} = x - c \Rightarrow c = -\frac{1}{4}$$

$$\frac{1}{4} + 4 \cdot \frac{1}{4} + \frac{1}{4} = 1$$

$$\textcircled{A} \quad \frac{x}{\sqrt{x^2-2}}$$

$y > 0 \leftarrow$ تابع باهشی و یک به یک

$$\sqrt{x^2-2} \rightarrow x^2-2 > 0 \rightarrow x^2 > 2 \rightarrow x > \sqrt{2} \quad \text{or} \quad x < -\sqrt{2}$$

$$D_f = (-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty)$$

~~این تابع را می توانیم به صورت زیر بنویسیم~~
 $y < 0 \leftarrow$ یک به یک ولی افزاینده

$$y = \frac{x}{\sqrt{x^2-2}} \Rightarrow y\sqrt{x^2-2} = x$$

$$y^2(x^2-2) = x^2 \rightarrow y^2x^2 - 2y^2 = x^2 \rightarrow x^2(y^2-1) = 2y^2$$

$$x^2 = \frac{2y^2}{y^2-1} \rightarrow x = \pm \sqrt{\frac{2y^2}{y^2-1}} \rightarrow \begin{matrix} y > 1 \rightarrow + \\ y < -1 \rightarrow - \end{matrix}$$

$$(4) f^{-1}(x) = \frac{x}{1+|x|} \quad g(x) = \sqrt{x} - 1$$

$$f\left(-\frac{r}{\omega}\right) + g^{-1}\left(-\frac{r}{\omega}\right)$$

$$x = f^{-1}(y) = \frac{y}{1+|y|}$$

$$f\left(-\frac{r}{\omega}\right) = \frac{-\frac{r}{\omega}}{1 - \frac{r}{\omega}} = \frac{-\frac{r}{\omega}}{\frac{\omega - r}{\omega}} = -\frac{r}{\omega - r}$$

$$y = \sqrt{x} - 1 \Rightarrow \sqrt{x} = y + 1 \Rightarrow x = (y + 1)^2$$

$$g^{-1}(x) = (x + 1)^2$$

$$g^{-1}\left(-\frac{r}{\omega}\right) = \left(-\frac{r}{\omega} + 1\right)^2 = \left(\frac{\omega - r}{\omega}\right)^2 = \frac{(\omega - r)^2}{\omega^2}$$

$$f\left(-\frac{r}{\omega}\right) + g^{-1}\left(-\frac{r}{\omega}\right) = -\frac{r}{\omega - r} + \frac{(\omega - r)^2}{\omega^2} = \boxed{-\frac{r\omega}{\omega^2}}$$

$$(1) F^{-1}(x) = \sqrt{x-1} \quad g(x) = f(x) + \sqrt{f(x)}$$

$$F^{-1}(x) = \sqrt{x-1}$$

$$y = f^{-1}(x) = \sqrt{x-1} \rightarrow x = f(y) = y^2 + 1$$

$$g(x) = f(x) + \sqrt{f(x)} = (x^2 + 1) + \sqrt{x^2 + 1}$$

$$g^{-1}(12) = x^2 + 1 + \sqrt{x^2 + 1} = 12$$

$$t = \sqrt{x^2 + 1} \Rightarrow x^2 + 1 = t^2 \rightarrow t^2 + t = 12 \rightarrow t^2 + t - 12 = 0$$

$$t = \frac{-1 \pm \sqrt{1 + 48}}{2} = \frac{-1 \pm 7}{2}$$

$$t = 3 \rightarrow t = \sqrt{x^2 + 1} \Rightarrow x^2 + 1 = 9 \Rightarrow x^2 = 8 \Rightarrow x = \pm 2\sqrt{2} \quad \left(\frac{1}{g(12)} = 2\right)$$