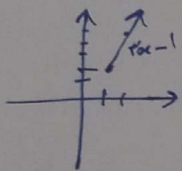


الف) $n^2 - n = 2 \Rightarrow n = 2 / -1 \rightarrow \text{مجموعه } (f^{-1} = \{ \dots (a, -1), \dots (2, -1) \}) \Rightarrow \boxed{n=2}$
 $\boxed{m=3}$

ب) $f = \{ (-1, 1), (1, 2), (2, 3), (a, \frac{1}{a-1}), (\frac{1}{a+2}, h), (m, 3) \}$
 $a = -1$ $\boxed{n=2}$ $m=2$



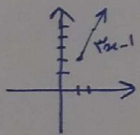
$$x + a < 2 \quad (x < 1) + a$$

$$x + a < 1 + a$$

$$\Downarrow$$

$$1 + a < 2$$

$$\boxed{a < 1}$$



$$ax + a - 1 < 2 \quad x \leq 0$$

$$ax \leq 0$$

$$ax + a \leq a$$

$$ax + a - 1 \leq -1 + a$$

$$\Downarrow$$

$$a - 1 < 2$$

$$\boxed{a < 3}$$

الف) $x^3 = y - 2 \Rightarrow f^{-1} = \sqrt[3]{x-2}$
 $x = \sqrt[3]{y-2}$

$y_1 = y_2$
 $x_1^3 + 2 = x_2^3 + 2 \Rightarrow x_1 = x_2$ تابع یک به یک است

ب) $y_1 = y_2$
 $(x_1 - 2)^3 = (x_2 - 2)^3 \Rightarrow x_1 = x_2$ تابع یک به یک است

$$y = (x-2)^3$$

$$y^3 = x-2$$

$$x = y^3 + 2 \Rightarrow f^{-1} = x^3 + 2$$

الف) $y_1 = y_2$
 $\frac{3x_1 + 1}{x_1 - 2} = \frac{3x_2 + 1}{x_2 - 2} \Rightarrow x_1 = x_2$ تابع یک به یک است

$$x = \frac{3y+1}{y-2}$$

$$xy - 2x = 3y + 1$$

$$xy - 3y = 2x + 1$$

$$\boxed{y = \frac{2x+1}{x-3}}$$

ب) $y = \frac{2(2+x)}{x-3} \Rightarrow y = 2$ تابع یک به یک نیست

الف) $y_1 = y_2$
 $\sqrt{x_1 - 3} = \sqrt{x_2 - 3} \Rightarrow x_1 = x_2$ تابع یک به یک است
 $y = \sqrt{x-3}$
 $y^2 = x-3$
 $x = y^2 + 3 \Rightarrow f^{-1} = x^2 + 3$

ب) $y = (x-3)(x-1)$
 $x = \frac{y}{x-3} \Rightarrow y = 0 \Rightarrow$ تابع یک به یک است

۶

$f(x) = y = x + \sqrt{x} \Rightarrow y = t^2 + t \quad t^2 + t - y = 0 \quad t = \frac{-1 \pm \sqrt{1+4y}}{2} \Rightarrow t = \frac{\sqrt{1+4y}-1}{2}$
 $x = \left(\frac{\sqrt{1+4y}-1}{2} \right)^2 = \frac{1+4y+1-2\sqrt{1+4y}}{4} = y + \frac{1}{4} - \frac{1}{2}\sqrt{1+4y}$
 $f^{-1} = x + \frac{1}{4} - \frac{1}{2}\sqrt{1+4x} = x + \frac{1}{4} - \sqrt{\frac{1}{4}(1+4x)} = x + \frac{1}{4} - \sqrt{x + \frac{1}{4}} = ax + b - \sqrt{x-c}$
 $a=1$
 $b=\frac{1}{4}$
 $c=-\frac{1}{4}$
 $a+2b+4c = 1 + \frac{1}{2} - 1 = \frac{1}{2}$

۷

$y_1 = y_2$
 $\frac{x_1}{\sqrt{x_1^2-2}} = \frac{x_2}{\sqrt{x_2^2-2}} \Rightarrow x_1 = x_2$ تابع یک به یک است
 $x = \frac{y}{\sqrt{y^2-2}} \quad x^2 = \frac{y^2}{y^2-2} \quad x^2 y^2 - 2x^2 = y^2$
 $x^2 y^2 - y^2 = 2x^2$
 $y^2(x^2-1) = 2x^2$
 $y^2 = \frac{2x^2}{x^2-1} \Rightarrow y = \frac{\sqrt{2}x}{\sqrt{x^2-1}}$

۸

$\frac{x}{1+|x|} = -\frac{2}{5} \begin{cases} \Rightarrow -2-2x = 5x \Rightarrow x = -\frac{2}{7} \text{ غلط} \\ \Rightarrow -2+2x = 5x \Rightarrow x = -\frac{2}{3} \text{ صحیح} \end{cases}$
 $\sqrt{x-1} = -\frac{2}{5}$
 $\sqrt{x} = \frac{2}{5}$
 $x = \frac{4}{25} \Rightarrow g^{-1} = \frac{4}{25}$
 $f(-\frac{2}{5}) + g^{-1}(-\frac{2}{5}) = -\frac{2}{5} + \frac{4}{25} = \frac{-23}{25}$

۹

$y = \sqrt{x-1}$
 $y^2 = x-1$
 $x = y^2 + 1 \Rightarrow f(x) = x^2 + 1 \Rightarrow g(x) = x^2 + 1 + \sqrt{x^2 + 1} = 12$
 $0 + \sqrt{0} = 12 \Rightarrow 0 = 4$
 $x^2 + 1 = 4$
 $x = 2$
 $g^{-1}(12) = 2$

۱۰