

الف) $f = \{(3, 2), (n, 5), (3, n^2 - n), (m, 2), (-1, 4)\}$
 $n^2 - n = 2 \quad n^2 - n - 2 = 0 \quad (n-2)(n+1) = 0 \rightarrow \boxed{n=2} \quad \text{یا} \quad n=-1 \quad \boxed{m=3}$

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ب) $f = \{(-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 3)\}$
 $m=2 \rightarrow a=-1 \quad \boxed{n=2} \quad \& \quad \{(-1, 1), (1, 2), (2, 3)\}$

$f(x) = \begin{cases} 3x-1 & x \geq 1 \\ x+a & x < 1 \end{cases} \rightarrow$

بالاترین نقطه ۲

$x+a \leq 2 \rightarrow 1+a \leq 2 \quad \boxed{a \leq 1} \quad (-\infty, 1]$

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$f(x) = \begin{cases} 3x-1 & x \geq 1 \\ ax+a-1 & x < 1 \end{cases}$
 $\hookrightarrow \textcircled{a} > 0 \quad a-1 < 2 \quad \textcircled{b} < 3 \rightarrow \boxed{0 < a < 3}$
 $\hookrightarrow a < 0 \quad a-1 > 2 \quad \textcircled{c} > 3$
 $\times$

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الف) $y = x^3 + 2 \quad \begin{cases} y = x_1^3 + 2 \\ y = x_2^3 + 2 \end{cases} \rightarrow x_1^3 + 2 = x_2^3 + 2 \quad x_1^3 = x_2^3 \quad x_1 = x_2 \quad \text{یک به یک است}$
 $\rightarrow x = y^{\frac{1}{3}} + 2 \quad y^{\frac{1}{3}} = x - 2 \quad y = \sqrt[3]{x-2}$

ب) $x^2 - 2x + 2$ سهمی - یک به یک نیست $\times$

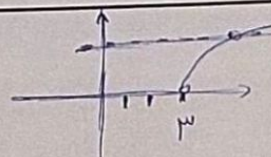
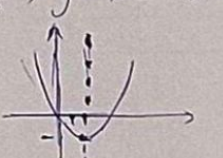
$\frac{-b}{2a} = 1 \quad y = -1$

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الف) $y = \frac{3x+1}{x-2}$ همگرا نیست یک به یک نیست $\checkmark$ است $x = \frac{3y+1}{y-2} \quad y(x-2) = 1+3x \quad y = \frac{3x+1}{x-2}$

ب) $y = \frac{4+2x}{x+2} \quad \frac{2(2+x)}{x+2} = 2 \quad x \neq -2$ تابع ثابت یک به یک نیست $\times$

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الف) $y = \sqrt{x-3}$ 	$f^{-1}(x) \Rightarrow x = \sqrt{y-3} \quad x^2 = y-3$ $y = x^2 + 3$ ب) $y = x^2 - 2x + 3 \quad x \in [2, +\infty)$  $x = y^2 - 2y + 3 \quad x = (y-1)^2 - 1 \quad y = 1 \pm \sqrt{x+1}$ $\underline{x=y} \Rightarrow f^{-1}(x) = 1 + \sqrt{x+1}$
	$f(x) = x + \sqrt{x} \quad y = x + \sqrt{x} \quad y = t^2 + t \quad t^2 + t - y = 0 \quad t = \frac{-1 \pm \sqrt{1+4y}}{2}$ $t = \sqrt{x} \rightarrow y = \left(\frac{-1 + \sqrt{1+4y}}{2} \right)^2 \rightarrow \frac{1+4x-2\sqrt{1+4x}+1}{4} = \frac{4x+2-2\sqrt{1+4x}}{4}$ $x + \frac{1}{y} - \left(\frac{1}{y} \right) \sqrt{1+4x} \quad \frac{ax+b-\sqrt{x-c}}{x+\frac{1}{y}-\sqrt{x+\frac{1}{y}}} \quad a=1 \quad b=\frac{1}{y} \quad c=-\frac{1}{y}$ $a+2b+fc = 1 + 2 \times \frac{1}{y} + \frac{1}{y} \times -\frac{1}{y} = \boxed{1}$
$y = \frac{x}{\sqrt{x^2-2}} \quad \begin{cases} \frac{x_1}{\sqrt{x_1^2-2}} = y \\ \frac{x_2}{\sqrt{x_2^2-2}} = y \end{cases} \quad \frac{x_1^2}{x_1^2-2} = \frac{x_2^2}{x_2^2-2} \quad x_1^2 x_2^2 - 2x_1^2 = x_1^2 x_2^2 - 2x_2^2 \quad x_1^2 = x_2^2 \quad \underline{\text{ملاحظة } x=y} \quad x_1 = x_2$ $f^{-1}(x) \Rightarrow x = \frac{y}{\sqrt{y^2-2}} \quad x\sqrt{y^2-2} = y \quad x^2 \left(\frac{y^2-2}{x^2} \right) = y^2 \quad y = \pm \frac{x}{x^2-x}$ $\underline{\text{ملاحظة } x=y} \quad y = + \frac{x}{x^2-x}$	$f^{-1}(x) = \frac{x}{1+ x } \quad \begin{cases} y = \frac{x}{1+x} & y+yx=x & y = \frac{x}{1-x} \quad \times \\ y = \frac{x}{1-x} & y-yx=x & y = \frac{x}{1+x} \quad \left(-\frac{r}{\omega} \right) \checkmark \end{cases}$ $g(x) = \sqrt{x} - 1 \quad \sqrt{x} = y - 1 \quad y = (x+1)^2$ $f\left(-\frac{r}{\omega}\right) + g\left(-\frac{r}{\omega}\right) \Rightarrow \frac{-\frac{r}{\omega}}{1-\frac{r}{\omega}} + \left(-\frac{r}{\omega} + 1\right)^2 = \frac{r}{\omega} + \frac{a}{r\omega} = \boxed{-\frac{r^2}{\sqrt{\omega}}}$
$f^{-1}(x) = \sqrt[3]{x-1} \quad y = x^3 + 1 \quad g(x) = f(x) + \sqrt{f(x)} \Rightarrow x^3 + 1 + \sqrt{x^3 + 1}$ $y = t^3 + 1 \quad t^3 + t - y = 0 \quad t = \frac{-1 \pm \sqrt{1+4y}}{2} \quad t = x^3 + 1 \quad x = \sqrt[3]{t-1}$ $x = \sqrt[3]{\left(\frac{-1 + \sqrt{1+4y}}{2} \right)^3 - 1} \xrightarrow{g^{-1}(12)} \sqrt[3]{\left(\frac{-1 + \sqrt{49}}{2} \right)^3 - 1} = \sqrt[3]{(3)^3 - 1} = \sqrt[3]{26} = \boxed{2}$	٩ ١٠