

الف) $f = \{(3, 2)(n, 5)(3, n^2 - n)(m, 2)(-1, 4)\}$

$n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow (n-2)(n+1) \rightarrow \begin{cases} n = -1 \\ n = 2 \end{cases}$

ب) $f = \{(-1, 1)(2, 2)(2, 3)(a, m-1)(a+2, n)(m, 2)\}$

$(-1, 1)(2, 2)(2, 3)(-1, 1)(2, 2)(2, 3)$

$(m=2) (n=2) (a=-1)$

$f(x) = \begin{cases} 2x-1; & x \geq 1 \\ x+a; & x < 1 \end{cases}$

$a=?$ ابتدا شرط داده باشد!

$f(1) = 2(1) - 1 = 1 \rightarrow R_1 = [1, +\infty)$

$R_2 = (-\infty, 1+a) \Rightarrow 1+a \leq 1 \Rightarrow a \leq 0$

$f(x) = \begin{cases} 2x-1; & x \geq 1 \\ ax+a-1; & x < 1 \end{cases}$

$a=?$

$ax+a-1 \rightarrow a > 0$

$y = 2x-1 \Rightarrow x \geq 1 \rightarrow 2(1)-1 = 1 \rightarrow y \geq 1$

$y = ax+a-1 = a-1 \rightarrow y \leq a-1$

$0 < a < 2$

الف) $y = x^2 + 2 \quad y_1 = y_2 \rightarrow x_1^2 + 2 = x_2^2 + 2 \rightarrow x_1 = x_2$

$x^2 = y - 2 \rightarrow x = \sqrt{y-2} \rightarrow y = \sqrt{x-2} \rightarrow f^{-1}(x) = \sqrt{x-2}$

ب) $y = x^2 - 2x + 2$

$y = x^2 - 2x + 2 = (x-1)^2 + 1$

الف) $y = \frac{2x+1}{x-2} \quad yx - 2y = 2x+1 \Rightarrow yx - 2x = y+1 \Rightarrow x(y-2) = y+1 \Rightarrow$

$x = \frac{y+1}{y-2} \rightarrow y = \frac{2x+1}{x-2} \Rightarrow f^{-1}(x) = \frac{2x+1}{x-2}$

ب) $y = \frac{4+2x}{x+2} \Rightarrow \frac{2(2+x)}{x+2} \Rightarrow y = 2$

$x = \frac{y+1}{y-2}$

$$a) y = \sqrt{x-2} \quad y_1 = y_1 \rightarrow \sqrt{x_1-2} = \sqrt{x_1-2} \rightarrow x_1 = x_1$$

$$(x) = \sqrt{y-2} \rightarrow x^2 = y-2 \quad y = x^2 + 2$$

$$b) y = x^2 - 2x + 2, x \in [2, +\infty)$$

$$y = x^2 - 2x + 2 + 1 - 1 \rightarrow y = x^2 - 2x + 1 - 1 \Rightarrow y = (x-1)^2 - 1 \Rightarrow (x-1)^2 = y+1 \Rightarrow$$

$$x-1 = \pm \sqrt{y+1} \Rightarrow x = 1 \pm \sqrt{y+1}$$

$$f^{-1}(x) = 1 + \sqrt{x+1}$$

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$$f(x) = x + \sqrt{x}$$

$$f^{-1}(x) = ax + b - \sqrt{x-c}$$

$$a + b + c = 1$$

$$t^2 + t - y = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1+4y}}{2} \Rightarrow t = \frac{-1 + \sqrt{1+4y}}{2}$$

$$f(x) = x + \sqrt{x} \rightarrow \sqrt{x} = t$$

$$f(x) = t^2 + t$$

$$\begin{cases} a=1 \\ b=\frac{1}{2} \\ c=-\frac{1}{4} \end{cases}$$

$$\sqrt{x} = \frac{-1 + \sqrt{1+4y}}{2} \Rightarrow x = \left(\frac{-1 + \sqrt{1+4y}}{2} \right)^2 \Rightarrow f^{-1}(x) = \left(\frac{-1 + \sqrt{1+4x}}{2} \right)^2$$

$$\Rightarrow \frac{1 + 1 + 4x - 2\sqrt{1+4x}}{4} = \frac{1}{2} + x - \frac{1}{2}\sqrt{1+4x} \Rightarrow x + \frac{1}{2} - \sqrt{\frac{1}{4} + x} = ax + b - \sqrt{x-c}$$

$$y = \frac{x}{\sqrt{x^2-2}} \rightarrow \frac{x}{\sqrt{x^2-2}} = \frac{x}{\sqrt{x^2-2}} \Rightarrow x^2 \cdot x^2 - 2x^2 = x^2 \cdot x^2 - 2x^2 \Rightarrow x^2 = x^2 \Rightarrow x_1 = x_1$$

$$f(x) = f(x_1) \Rightarrow y = \frac{x}{\sqrt{x^2-2}} \Rightarrow x = \frac{y}{\sqrt{y^2-2}} \Rightarrow x^2 = \frac{y^2}{y^2-2} \Rightarrow x^2 y^2 - 2x^2 = y^2 \Rightarrow$$

$$y^2 x^2 - 2y^2 = y^2 \Rightarrow y^2 (x^2 - 2) = y^2 \Rightarrow y^2 = \frac{2x^2}{x^2-2} \Rightarrow y = \pm \frac{\sqrt{2x^2}}{x^2-2} \Rightarrow y = \frac{\sqrt{2x^2}}{x^2-2}$$

$$f^{-1}(x) = \frac{x}{1+|x|} \quad g^{-1}\left(-\frac{1}{2}\right) + f\left(-\frac{1}{2}\right) = 0$$

$$g(x) = \sqrt{x-1} \quad -\frac{1}{2} = \frac{x}{1+|x|} \Rightarrow x = -1 + 2x \Rightarrow x = -1 \Rightarrow x = -\frac{1}{2}$$

$$\frac{1}{2} - \frac{1}{2} = \frac{-1}{2}$$

$$\sqrt{x-1} = -\frac{1}{2} \Rightarrow (\sqrt{x-1})^2 = \left(-\frac{1}{2}\right)^2 \Rightarrow x-1 = \frac{1}{4} \Rightarrow x = \frac{5}{4}$$

$$f^{-1}(x) = \sqrt{x-1}$$

$$g(x) = f(x) + \sqrt{f(x)} \quad x = \sqrt{y-1} \Rightarrow x^2 = y-1 \Rightarrow y = x^2 + 1$$

$$g(x) = x^2 + 1 + \sqrt{x^2 + 1}$$

$$g^{-1}(1) = 0$$

$$t^2 + t = 1 \Rightarrow t^2 + t - 1 = 0 \Rightarrow (t+1)(t-1) = 0$$

$$g(1) = 1 \quad x^2 + 1 = t^2 \quad \frac{x^2 + 1}{t} = t$$

$$(1)^2 = (\sqrt{x^2+1})^2 \Rightarrow 1 = x^2 + 1 \Rightarrow x = 0$$

$$x = 0$$