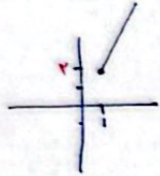


الف) $n^2 - n = 2 \Rightarrow n \in \mathbb{Z}$, $m = 3$

ب) $m = 2 \Rightarrow a = -1 \Rightarrow n = 2$



$x = 1 \rightarrow f(x) = 2$

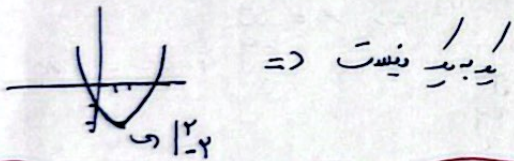
$x + a \leq 2 \Rightarrow a \leq 1$

چون داده $a < 1$ پس اشتباه نماند

$x = 1 \rightarrow f(x) = 2 - 1 = 1 \Rightarrow a \cdot x + a - 1 < 2 \Rightarrow a - 1 < 2 \Rightarrow a < 3 \rightarrow 0 < a < 3$

الف) $y = x^3 + 2$ $\Rightarrow$ یک به یک است $x^3 = y - 2 \Rightarrow x = \sqrt[3]{y - 2} \Rightarrow f^{-1}(x) = \sqrt[3]{x - 2}$

ب) $y = x^2 - 4x + 2 = x^2 - 4x + 4 - 2 = (x - 2)^2 - 2$



الف) $y = \frac{x + 1}{x - 2}$ $\Rightarrow$ یک به یک است و هموار است $x = -\frac{(-y - 1)}{y - 3} = \frac{y + 1}{y - 3} \Rightarrow f^{-1}(x) = \frac{x + 1}{x - 3}$

ب) $y = \frac{4 + 2x}{x + 2} = \frac{2(2 + x)}{x + 2} = 2 \Rightarrow$ یک به یک نیست $x \neq -2$

الف) $y = \sqrt{x - 3}$ $y^2 = x - 3 \Rightarrow x = y + 3 \rightarrow y = x + 3$ $\Rightarrow$ یک به یک است

ب) $y = x^2 - 4x + 3 = (x - 2)^2 - 1$ $y + 1 = (x - 2)^2 \Rightarrow x - 2 = \pm \sqrt{y + 1} \Rightarrow x = 2 \pm \sqrt{y + 1}$
 $y = 2 \pm \sqrt{x + 1}$
 غنای $(y \geq -1)$

$$y = x + \sqrt{x} \rightarrow y = (\sqrt{x + \frac{1}{y}})^2 - \frac{1}{y} \quad (\sqrt{x + \frac{1}{y}})^2 = y + \frac{1}{y} \quad \sqrt{x + \frac{1}{y}} = \sqrt{y + \frac{1}{y}} \quad -V$$

$$\sqrt{x} = \frac{1}{y} + \sqrt{y + \frac{1}{y}} \rightarrow x = \frac{1}{y^2} - \sqrt{y + \frac{1}{y}} + y + \frac{1}{y} = y + \frac{1}{y} - \sqrt{y + \frac{1}{y}}$$

$$f(x)^{-1} = \frac{1}{y} - \sqrt{x + \frac{1}{y}} + x \quad \begin{matrix} a=1 \\ b=\frac{1}{y} \\ c=-\frac{1}{y} \end{matrix} \left\{ \begin{matrix} a + 2b + c = 1 + 1 - 1 = 1 \end{matrix} \right.$$

$$y = \frac{x}{\sqrt{x^2 - 2}} \quad y_1 = y_2 \Rightarrow \frac{x_1}{\sqrt{x_1^2 - 2}} = \frac{x_2}{\sqrt{x_2^2 - 2}} \rightarrow \frac{x_1^2}{x_1^2 - 2} = \frac{x_2^2}{x_2^2 - 2} \quad -A$$

$$x_1^2 (x_2^2 - 2) = x_2^2 (x_1^2 - 2)$$

$$x_1^2 x_2^2 - 2x_1^2 = x_2^2 x_1^2 - 2x_2^2 \rightarrow x_1^2 = x_2^2 \quad x_1 = \pm x_2$$

$$f(x)^{-1} = y^2 = \frac{x^2}{x^2 - 2}$$

$$x^2 = \frac{(-2y^2)}{y^2 - 1} \Rightarrow x = \frac{|y|\sqrt{2}}{\sqrt{y^2 - 1}}$$

علامت x و y یکی است $\Leftarrow$ یکی $-$ یکی $+$ است

$$F(x)^{-1} = \frac{\sqrt{2}x}{\sqrt{x^2 - 1}}$$

$$f^{-1}(x) = \frac{x}{1+|x|} \quad \begin{matrix} x > 0 \rightarrow y + yx = x \Rightarrow y = x(1-y) \Rightarrow x = \frac{y}{1-y} \\ x < 0 \rightarrow y - yx = x \Rightarrow x = \frac{y}{1+y} \end{matrix} \left\{ \begin{matrix} f(x) = \frac{x}{1-|x|} \end{matrix} \right. \quad -9$$

$$g(x) = \sqrt{x} - 1 \quad y + 1 = \sqrt{x} \quad x = y^2 + 2y + 1 \rightarrow g^{-1}(x) = (x+1)^2$$

$$f\left(-\frac{r}{\delta}\right) + g^{-1}\left(-\frac{r}{\delta}\right) = \frac{-\frac{r}{\delta}}{1-\frac{r}{\delta}} + \left(1 - \frac{r}{\delta}\right)^2 = \frac{-r^2}{\delta^2}$$

$$f^{-1}(x) = \sqrt[3]{x-1} \quad y = \sqrt[3]{x-1} \quad x = \sqrt[3]{y-1} \quad x^3 = y-1 \quad y = x^3 + 1 \quad f^{-1}(x) = x^3 + 1 \quad -10$$

$$g(x) = f(x) + \sqrt{f(x)} = x^3 + 1 + \sqrt{x^3 + 1} = (\sqrt{x^3 + 1} + \frac{1}{\sqrt{x^3 + 1}})^2 - \frac{1}{x^3 + 1} \rightarrow \sqrt{y + \frac{1}{y}} = \sqrt{x^3 + 1} + \frac{1}{\sqrt{x^3 + 1}}$$

$$\hookrightarrow = 12 \rightarrow \frac{12}{5}$$

$$\sqrt{x^3 + 1} = \sqrt{y + \frac{1}{y}} - \frac{1}{\sqrt{y + \frac{1}{y}}} \quad x^3 = y + \frac{1}{y} - \sqrt{y + \frac{1}{y}}$$

$$g^{-1}(x) = \sqrt[3]{x + \frac{1}{y}} - \sqrt{x + \frac{1}{y}} \rightarrow g^{-1}(x) = \sqrt[3]{12 + \frac{1}{y}} - \sqrt{\frac{12}{y}} = \sqrt[3]{12 + \frac{1}{y}} - \frac{\sqrt{12}}{\sqrt{y}} = \sqrt[3]{4} = \sqrt[3]{\frac{64}{8}} = \sqrt[3]{\frac{64}{8}} = \sqrt[3]{8} = 2$$

$$f(x) + \sqrt{f(x)} = 12 \rightarrow \sqrt{f(x)} = 3 \rightarrow f(x) = 9 \quad g^{-1}(12) = 2$$