

الف) $f = \{(3,2), (n,5), (3, n-2), (m,2), (-1,4)\}$ چون $(3, n-2) \xrightarrow{if n=2} (3,2) = (m,2)$
 $n-2 = 2 \rightarrow n-2-2=0 \rightarrow (n-2)(n+1) \rightarrow n=2 \text{ یا } -1$ نقطه $(n=2, m=3)$

ب) $f = \{(-1,1), (1,2), (2,3), (a, m-1), (a+2, n), (m, 3)\}$
 $(-1,1) = (a,1) \rightarrow a = -1$
 $(2,3) = (m,3) \xrightarrow{m=2} (-1,2) = (1,2) = (1,n) \Rightarrow n=2$

$f(x) = \begin{cases} 3x-1 & ; x \geq 1 \\ x+a & ; x < 1 \end{cases} \rightarrow \begin{cases} \text{if } x=1 \Rightarrow y=2 \\ \text{if } x=2 \Rightarrow y=5 \end{cases}$
 $\rightarrow \text{if } x=1 \Rightarrow y=1+a$
 $y=2 \Rightarrow a=1$
 $a \in (-\infty, 1]$

$f(x) = \begin{cases} 3x-1 & ; x \geq 1 \\ ax+a-1 & ; x < 0 \end{cases} \rightarrow \begin{cases} \text{if } x=1 \Rightarrow y=2 \\ \text{if } x=0 \Rightarrow y=a-1 \end{cases}$
 $a-1 < 2 \rightarrow a < 3$
 $a < 3 \Rightarrow a \in (-\infty, 3)$

الف) $y = x^3 + 2$ $y_1 = y_2 \rightarrow x_1^3 + 2 = x_2^3 + 2 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$
 $f^{-1}(x) \Rightarrow y-2 = x^3 \rightarrow x = \sqrt[3]{y-2} \Rightarrow y = \sqrt[3]{x-2}$
 ب) $y = x^2 - 4x + 2 \rightarrow y_1 = y_2 \Rightarrow (x_1-2)^2 - 2 = (x_2-2)^2 - 2 \Rightarrow (x_1-2)^2 = (x_2-2)^2$
 $x_1-2 = |x_2-2|$
 (دو خط متوازی دراصل یک خط هستند، در نتیجه یک به یک نیست و دایره داشته باشد)

الف) $y = \frac{3x+1}{x-2}$ $\rightarrow$ $y(x-2) = 3x+1$
 $yx - 2y = 3x+1$
 $yx - 3x = 1+2y$
 $x(y-3) = 1+2y \Rightarrow x = \frac{1+2y}{y-3} \Rightarrow f^{-1}(x) = y = \frac{1+2x}{x-3}$
 ب) $y = \frac{4+2x}{x+2}$
 $y = \frac{2x+4}{x+2} = \frac{2(x+2)}{x+2} = 2$
 (توجه: این تابع یک خط افقی است)

الف) $y = \sqrt{x-1}$ $y_1 = y_2 \rightarrow \sqrt{x_1-1} = \sqrt{x_2-1} \xrightarrow{\text{مربع}} x_1-1 = x_2-1 \Rightarrow x_1 = x_2$
 $f^{-1}(x) \rightarrow y^2 = x-1 \Rightarrow x = y^2+1 \Rightarrow y = \sqrt{x-1}$

ب) $y = x^2 - 4x + 1, x \in [1, +\infty)$
 $x_5 = \frac{r}{1} = r$

$f^{-1}(x) \rightarrow y = (x-1)^2 - 1 \Rightarrow y+1 = (x-1)^2 \Rightarrow \sqrt{y+1} = x-1 \Rightarrow x = \sqrt{y+1} + 1$
 $y = \sqrt{x+1} + 1$

$f(x) = x + \sqrt{x}$ $f^{-1}(x) = ax + b - \sqrt{x-c}$ $a + rb + \epsilon c = ?$
 $\sqrt{x} = t \Rightarrow y = t^2 + t \rightarrow t^2 + t - y = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1+4y}}{2}$, $\sqrt{x} = t \Rightarrow x = t^2 \Rightarrow t = \left(\frac{-1 \pm \sqrt{1+4y}}{2} \right)^2$
 $x \in \mathbb{R}^+ \rightarrow \frac{1 + \sqrt{1+4y} - 2\sqrt{1+4y}}{2} \Rightarrow \frac{1 - \sqrt{1+4y}}{2} \rightarrow y + \frac{1}{r} - \frac{1}{r}\sqrt{1+4y} \Rightarrow f^{-1}(x) = x + \frac{1}{r} - \frac{1}{r}\sqrt{1+4x}$
 $x + \frac{1}{r} - \frac{1}{r}\sqrt{1+4x} = ax + b - \sqrt{x-c} \Rightarrow \underline{a=1}, \underline{b=\frac{1}{r}}$
 $-\frac{1}{r}\sqrt{1+4x} = -\sqrt{x-c} \xrightarrow{\text{مربع}} \frac{1}{r^2}(1+4x) = x-c \xrightarrow{\text{ضرب في } r^2} 1+4x = rx - rc \Rightarrow -rc = 1 \Rightarrow c = -\frac{1}{r}$
 $a + rb + \epsilon c = 1 + r \cdot \frac{1}{r} - 1 = 1$
 $a=1, b=\frac{1}{r}, c=-\frac{1}{r}$

$y = \frac{x}{\sqrt{x^2-1}}$

$y_1 = y_2 \Rightarrow \frac{x_1}{\sqrt{x_1^2-1}} = \frac{x_2}{\sqrt{x_2^2-1}} \xrightarrow{\text{مربع}} \frac{x_1^2}{x_1^2-1} = \frac{x_2^2}{x_2^2-1}$
 $\frac{x_1^2}{x_1^2-1} = \frac{x_2^2}{x_2^2-1} \Rightarrow x_1^2(x_2^2-1) = x_2^2(x_1^2-1) \Rightarrow x_1^2 x_2^2 - x_1^2 = x_1^2 x_2^2 - x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

$f^{-1}(x) \rightarrow y = \frac{x}{\sqrt{x^2-1}} \rightarrow x = \frac{y}{\sqrt{y^2-1}} \Rightarrow x^2 = \frac{y^2}{y^2-1} \Rightarrow x^2(y^2-1) = y^2 \Rightarrow x^2 y^2 - x^2 = y^2 \Rightarrow x^2 y^2 - y^2 = x^2 \Rightarrow y^2(x^2-1) = x^2 \Rightarrow y = \pm \sqrt{\frac{x^2}{x^2-1}}$
 $y = \frac{x}{\sqrt{x^2-1}}$

$f^{-1}(x) = \frac{x}{1+|x|}$, $g(x) = \sqrt{x} - 1$ $f\left(-\frac{r}{\delta}\right) + g^{-1}\left(-\frac{r}{\delta}\right)$
 $g^{-1}\left(-\frac{r}{\delta}\right) \xrightarrow{\text{مربع}} g^{-1}\left(-\frac{r}{\delta}\right) = m \Rightarrow g(m) = -\frac{r}{\delta} \Rightarrow \sqrt{x} - 1 = -\frac{r}{\delta} \Rightarrow \sqrt{x} = -\frac{r}{\delta} + 1 \Rightarrow \frac{-4\delta}{\delta} = \frac{r}{\delta} \Rightarrow x = \frac{r^2}{4\delta^2}$
 $f\left(-\frac{r}{\delta}\right) + g^{-1}\left(-\frac{r}{\delta}\right) = -\frac{r}{1+\frac{r}{\delta}} + \frac{r}{4\delta^2} = \frac{-r\delta + r}{\delta(1+\frac{r}{\delta})} = \frac{-r\delta + r}{\delta + r}$
 $\frac{x}{1+|x|} = -\frac{r}{\delta} \xrightarrow{\text{if } x = -\frac{r}{\delta}} \frac{-\frac{r}{\delta}}{1+\frac{r}{\delta}} = \frac{-\frac{r}{\delta}}{\frac{\delta+r}{\delta}} = \frac{-r}{\delta+r} \Rightarrow f\left(-\frac{r}{\delta}\right) = -\frac{r}{\delta+r}$

$f^{-1}(x) = \sqrt{x-1}$ $g(x) = f(x) + \sqrt{f(x)}$ $g^{-1}(12) = ?$

$2 f(x) \rightarrow y = \sqrt{x-1} \xrightarrow{\text{مربع}} y^2 = x-1 \Rightarrow y^2+1 = x \Rightarrow y = \sqrt{x-1}$

$g(x) = x^2 + 1 + \sqrt{x^2+1}$

$g^{-1}(12) \rightarrow x^2 + 1 + \sqrt{x^2+1} = 12 \xrightarrow{\text{مربع}} \text{if } x=2 \Rightarrow 4+1+\sqrt{4+1} = 12 \Rightarrow g^{-1}(12) = 2$