

الف) $f = \{(3,2), (n,5), (3, n^2-n), (m,2), (-1,4)\}$

$n^2-n=2 \rightarrow n^2-n-2=0 \rightarrow (n-2)(n+1) \rightarrow n=2, -1$

ب) $f = \{(-1,1), (1,2), (2,3), (a, m-1), (a+2, n), (m, 3)\}$

$(-1,1) = (a,1) \rightarrow a = -1$

$(2,3) = (m,3) \rightarrow m=2$

$(1,2) = (a+2, n) \Rightarrow (1,2) = (1,n) \Rightarrow n=2$

چون $(3, n^2-n) \xrightarrow{if n=2} (3,2) = (m,2)$

نقطه $(n=2, m=3)$

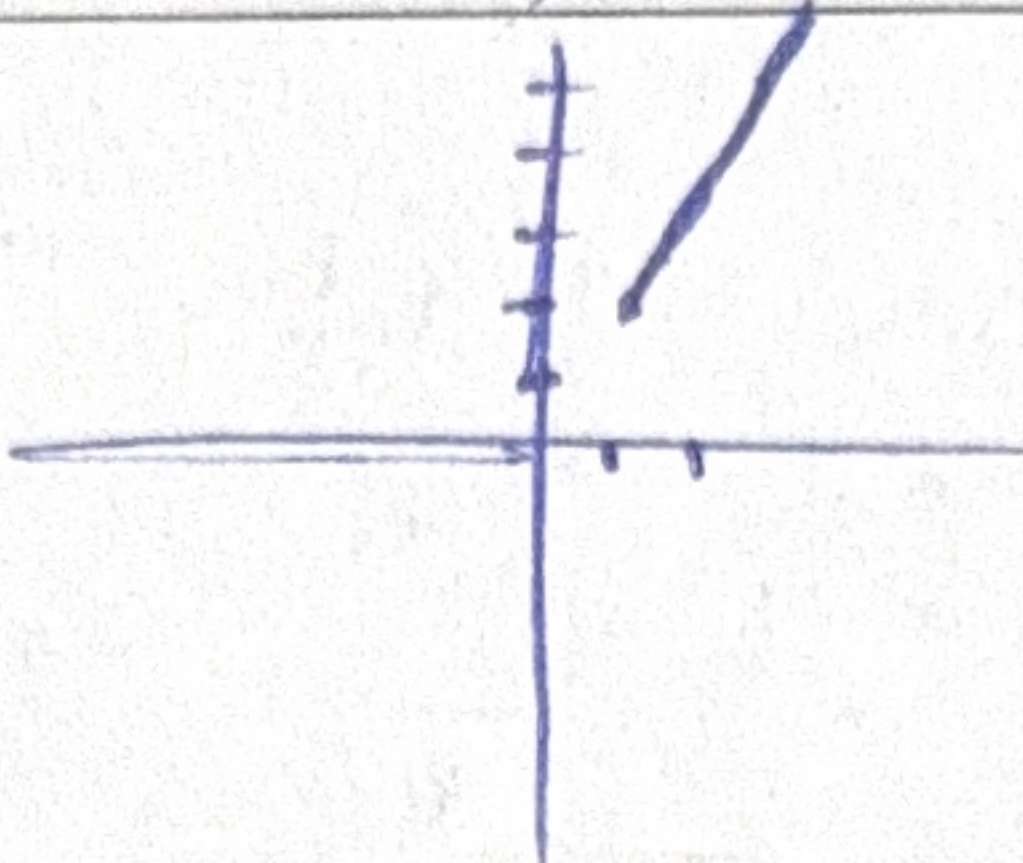
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$f(x) = \begin{cases} 3x-1 & ; x \geq 1 \\ x+a & ; x < 1 \end{cases} \rightarrow \begin{cases} if x=1 \Rightarrow y=2 \\ if x=2 \Rightarrow y=5 \end{cases}$

$\hookrightarrow if x=1 \Rightarrow y=1+a$

$y=2 \Rightarrow a=1$

$a \in (-\infty, 1]$



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$f(x) = \begin{cases} 3x-1 & ; x \geq 1 \\ ax+a-1 & ; x < 0 \end{cases} \rightarrow \begin{cases} if x=1 \Rightarrow y=2 \\ if x=0 \Rightarrow y=a-1 \end{cases}$

$a-1 < 2 \rightarrow a < 3$

$a < 3 \Rightarrow a \in (-\infty, 3)$

تابع قطعه ای

در هر دو بخش تابع

نقطه ثابت

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الف) $y = x^3 + 2$

$f^{-1}(x) \Rightarrow y-2 = x^3 \rightarrow x = \sqrt[3]{y-2} \Rightarrow y = \sqrt[3]{x-2}$

ب) $y = x^2 - 4x + 2 \rightarrow y_1 = y_2 \Rightarrow (x_1-2)^2 - 2 = (x_2-2)^2 - 2 \Rightarrow (x_1-2)^2 = (x_2-2)^2$

$x_1-2 = |x_2-2|$

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الف) $y = \frac{3x+1}{x-2}$

$y(x-2) = 3x+1$

$yx - 2y = 3x+1$

$x(y-3) = 1+2y \Rightarrow x = \frac{1+2y}{y-3} \Rightarrow f^{-1}(x) = y = \frac{1+2x}{x-3}$

ب) $y = \frac{4+2x}{x+2}$

$y = \frac{2x+4}{x+2} = \frac{2(x+2)}{x+2} = 2$

$y=2$

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الف) $y = \sqrt{x-2}$ $y_1 = y_2 \rightarrow \sqrt{x_1-2} = \sqrt{x_2-2} \xrightarrow{\text{مربع}} x_1-2 = x_2-2 \Rightarrow x_1 = x_2$
 $f^{-1}(x) \rightarrow y^2 = x-2 \Rightarrow x = y^2+2 \Rightarrow y = \sqrt{x-2}$

ب) $y = x^2 - 4x + 2, x \in [2, +\infty)$

$x_5 = \frac{r}{1} = r$

$f^{-1}(x) \rightarrow y = (x-2)^2 - 1 \Rightarrow y+1 = (x-2)^2 \Rightarrow \sqrt{y+1} = x-2 \Rightarrow x = \sqrt{y+1} + 2$
 $y = \sqrt{x+1} + 2$

$f(x) = x + \sqrt{x}$ $f^{-1}(x) = ax + b - \sqrt{x-c}$ $a + rb + \epsilon c = ?$
 $\sqrt{x} = t \Rightarrow y = t^2 + t \rightarrow t^2 + t - y = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1+4y}}{2}$, $\sqrt{x} = t \Rightarrow x = t^2 \Rightarrow t = \left(\frac{-1 \pm \sqrt{1+4y}}{2} \right)^2$
 $x \in \mathbb{R}^+ \rightarrow \frac{1 + \sqrt{1+4y}}{2} = t \Rightarrow \frac{1 + \sqrt{1+4y}}{2} = \sqrt{x} \Rightarrow y + \frac{1}{4} - \frac{1}{4}\sqrt{1+4y} = f^{-1}(x) = x + \frac{1}{4} - \frac{1}{4}\sqrt{1+4x}$
 $x + \frac{1}{4} - \frac{1}{4}\sqrt{1+4x} = ax + b - \sqrt{x-c} \Rightarrow a=1, b=\frac{1}{4}$
 $-\frac{1}{4}\sqrt{1+4x} = -\sqrt{x-c} \xrightarrow{\text{مربع}} \frac{1}{16}(1+4x) = x-c \xrightarrow{\text{ضرب في 4}} 1+x = 4x-4c \Rightarrow 1+x = 4x-4c \Rightarrow -3x = -4c-1 \Rightarrow c = \frac{3x+1}{4}$
 $a+rb+\epsilon c = 1 + \frac{1}{4} + \frac{1}{4} = \frac{5}{4}$

$y = \frac{x}{\sqrt{x^2-1}}$

$y_1 = y_2 \Rightarrow \frac{x_1}{\sqrt{x_1^2-1}} = \frac{x_2}{\sqrt{x_2^2-1}} \xrightarrow{\text{مربع}} \frac{x_1^2}{x_1^2-1} = \frac{x_2^2}{x_2^2-1}$

$\frac{x_1^2}{x_1^2-1} = \frac{x_2^2}{x_2^2-1} \Rightarrow x_1^2(x_2^2-1) = x_2^2(x_1^2-1) \Rightarrow x_1^2 x_2^2 - x_1^2 = x_1^2 x_2^2 - x_2^2 \Rightarrow -x_1^2 = -x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

$f^{-1}(x) \rightarrow y = \frac{x}{\sqrt{x^2-1}} \rightarrow x = \frac{y}{\sqrt{y^2-1}} \Rightarrow x^2 = \frac{y^2}{y^2-1} \Rightarrow x^2(y^2-1) = y^2 \Rightarrow x^2 y^2 - x^2 = y^2 \Rightarrow x^2 y^2 - y^2 = x^2 \Rightarrow y^2(x^2-1) = x^2 \Rightarrow y = \pm \sqrt{\frac{x^2}{x^2-1}}$

$f^{-1}(x) = \frac{x}{1+|x|}$, $g(x) = \sqrt{x} - 1$ $f\left(-\frac{r}{\delta}\right) + g^{-1}\left(-\frac{r}{\delta}\right)$
 $g^{-1}\left(-\frac{r}{\delta}\right) \xrightarrow{\text{مربع}} g^{-1}\left(-\frac{r}{\delta}\right) = m \Rightarrow g(m) = -\frac{r}{\delta} \Rightarrow \sqrt{m} - 1 = -\frac{r}{\delta} \Rightarrow \sqrt{m} = -\frac{r}{\delta} + 1 \Rightarrow \sqrt{m} = \frac{-r + \delta}{\delta} \Rightarrow m = \frac{(-r + \delta)^2}{\delta^2}$

$f^{-1}(x) = \frac{x}{1+|x|}$ $f\left(-\frac{r}{\delta}\right) + g^{-1}\left(-\frac{r}{\delta}\right) = \frac{-\frac{r}{\delta}}{1+\frac{r}{\delta}} + \frac{(-r + \delta)^2}{\delta^2} = \frac{-r}{\delta + r} + \frac{(-r + \delta)^2}{\delta^2}$

$f^{-1}(x) = \sqrt{x-1}$ $g(x) = f(x) + \sqrt{f(x)}$ $g^{-1}(12) = ?$

$2 f(x) \rightarrow y = \sqrt{x-1} \xrightarrow{\text{مربع}} y^2 = x-1 \Rightarrow y^2+1 = x \Rightarrow y = \sqrt{x-1}$

$g(x) = x^2 + 1 + \sqrt{x^2 + 1}$

$g^{-1}(12) \rightarrow x^2 + 1 + \sqrt{x^2 + 1} = 12 \xrightarrow{\text{مربع}} \text{if } x=2 \Rightarrow 4+1+\sqrt{4+1} = 12 \Rightarrow g^{-1}(12) = 2$