

ا) $y^r = x - r \rightarrow x = y^r + r \rightarrow y = x^r + r \rightarrow D_{f^{-1}} = [0, +\infty)$

ب) $y = x^r - (x + r) = (x - r)^r - 1 \rightarrow \sqrt{y + 1} = x - r \rightarrow y = \sqrt{x + 1} + r, D_{f^{-1}} = [-1, +\infty)$

$y = (x + \sqrt{x} + \frac{1}{x}) - \frac{1}{x} \rightarrow y = (\sqrt{x} + \frac{1}{x})^2 - \frac{1}{x} \rightarrow \sqrt{y + \frac{1}{x}} = \sqrt{x} + \frac{1}{x}$

$\sqrt{x} = \sqrt{y + \frac{1}{x}} - \frac{1}{x} \rightarrow x = y + \frac{1}{x} + \frac{1}{x} - \sqrt{y + \frac{1}{x}} \rightarrow y = x + \frac{1}{x} - \sqrt{x + \frac{1}{x}} = ax + b\sqrt{x} + c$

$a = 1, b = \frac{1}{x}, c = -\frac{1}{x} \Rightarrow 1 + 1 - 1 = 1$

$\left| \frac{x_1}{y} \right| \left| \frac{x_2}{y} \right| \rightarrow \frac{x_1}{\sqrt{x_1^2 - r}} = \frac{x_2}{\sqrt{x_2^2 - r}} \rightarrow \frac{x_1^r}{x_1^r - r} = \frac{x_2^r}{x_2^r - r} \rightarrow x_1^r x_2^r - r x_1^r = x_1^r x_2^r - r x_2^r$

$x_1 = x_2 \rightarrow \frac{x}{\sqrt{x^2 - r}} = \frac{x}{\sqrt{x^2 - r}} \rightarrow y^r = \frac{x}{\sqrt{x^2 - r}} \rightarrow x^r y^r - r y^r = x^r + x^r y^r - r y^r = r y^r$

$x^r (y^r - 1) = r y^r \rightarrow x^r = \frac{r y^r}{y^r - 1} \rightarrow x = \sqrt[r]{\frac{r y^r}{y^r - 1}} \rightarrow x = \frac{\sqrt[r]{r y}}{\sqrt[r]{y^r - 1}}$

$g(x) = \sqrt{x} - 1 = -\frac{r}{\omega} \rightarrow \sqrt{x} = \frac{r}{\omega} \rightarrow x = \frac{r^2}{\omega^2} \rightarrow g^{-1}(-\frac{r}{\omega}) = \frac{r^2}{\omega^2}$

$f(x) = \frac{x}{1 + |x|} = -\frac{r}{\omega} \rightarrow -r - r|x| = \omega x \rightarrow \omega x + r|x| + r = 0 \rightarrow \forall x + r = 0 \rightarrow x = -\frac{r}{\omega}$

$\rightarrow x = -\frac{r}{\omega} \rightarrow \frac{-\frac{r}{\omega}}{1 + |-\frac{r}{\omega}|} = -\frac{r}{\omega} \times$

$\rightarrow x = -\frac{r}{\omega} \rightarrow \frac{-\frac{r}{\omega}}{1 + |-\frac{r}{\omega}|} = -\frac{r}{\omega} \checkmark \sim \neq (-\frac{r}{\omega}) = -\frac{r}{\omega} \Rightarrow -\frac{r}{\omega} + \frac{r}{\omega} = \frac{r}{\omega}$

$g(x) = \lim_{x \rightarrow 9} \sqrt{x} = 12 \rightarrow \lim_{x \rightarrow 9} = 9 \rightarrow f^{-1}(9) = \sqrt{9 - 1} = 2$

$g^{-1}(12) = 2$