

۱.

$$(A \times B) - (B \times B) \rightarrow \{(1,2), (1,3), (2,2), (2,3), (3,2), (3,3)\} - \{(2,2), (2,3), (3,2), (3,3)\}$$

$$\Rightarrow (A \times B) - B^2 = \{(1,2), (1,3)\}$$

۲.

الف) $f(x) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$

در صورتی تابع خواهد بود که برای هر x انتخاب مقدار داشته باشیم

$$m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow (m-2)(m+1) = 0$$

$$\begin{cases} m-2=0 \rightarrow m=2 \\ m+1=0 \rightarrow m=-1 \end{cases}$$

بررسی مقادیر m بدست آمده

$m=2 \rightarrow \{(3, 4), (2, 1), (-3, 2), (-2, 2), (3, 4), (2, 4)\}$ غرق

به ازای $x=2$ دو مقدار داریم $\Rightarrow$ غیر قابل قبول

$m=-1 \rightarrow \{(3, 1), (2, 1), (-3, -1), (-2, -1), (3, 1), (-1, 4)\}$ قق

تابع $f(x)$ نخواهد بود برای $m=-1$ قابل قبول نباشد

$g(x) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$

$$m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow (m-2)(m+1) = 0$$

$$\begin{cases} m-2=0 \rightarrow m=2 \\ m+1=0 \rightarrow m=-1 \end{cases}$$

بررسی مقادیر m

if $m=2 \rightarrow \{(3, 4), (2, 1), (2, 4), (3, 4), (-2, 2), (1, 3)\}$

به ازای $x=2$ دو مقدار داریم $\Rightarrow$ غیر قابل قبول

if $m=-1 \rightarrow \{(3, 1), (2, 1), (-1, 4), (3, 1), (-2, -1), (-1, -1)\}$

به ازای $x=3$ دو مقدار داریم $\Rightarrow$ غیر قابل قبول

تابع $g(x)$ به ازای هیچ یک از مقادیر m قابل قبول نمی باشد

۳.

$$f(x) = \begin{cases} x^2-1 & x < 1 \\ ax+b & 1 \leq x \leq 2 \\ x^3 & x \geq 2 \end{cases}$$

$x=1 \Rightarrow a \cdot 1 + b = 1^2 - 1 \rightarrow a + b = 0$

$x=2 \Rightarrow a \cdot 2 + b = 2^3 \rightarrow 2a + b = 8$

$$\begin{cases} a+b=0 \rightarrow a=-b \\ 2a+b=8 \end{cases} \rightarrow 2(-b) + b = 8 \rightarrow -2b + b = 8 \rightarrow -b = 8 \rightarrow b = -8$$

$$b = -8 \xrightarrow{a = -b} a = -(-8) \Rightarrow a = 8 \Rightarrow a \times b = -8 \times 8 = -64$$

$$f(x) = \begin{cases} 2x-3 & x > k \\ k-3x & x \leq k \end{cases} \xrightarrow{\text{if}} x=k \Rightarrow 2k-3 = k-3k \rightarrow \Delta k = \frac{3}{2}$$

$$\Rightarrow \boxed{k = \frac{3}{2}}$$

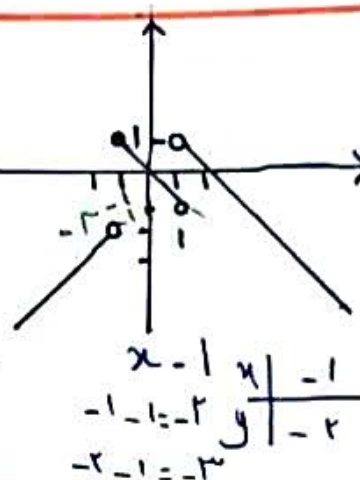
د.

$$f(x) = \begin{cases} -x+2 & x > 1 \\ -x & -1 < x < 1 \\ x-1 & x < -1 \end{cases}$$

$$\begin{aligned} -x+2 & \\ -1+2 &= 1 \\ -2+2 &= 0 \end{aligned}$$

$$\frac{x}{y} \begin{array}{c|c} 1 & 2 \\ \hline 1 & 0 \end{array}$$

$$\begin{aligned} -x & \\ -(-1) &= 1 \\ -(1) &= -1 \end{aligned} \quad \frac{x}{y} \begin{array}{c|c} -1 & 1 \\ \hline 1 & -1 \end{array}$$



$$D_f = (-\infty, 1) \cup (1, +\infty) \\ R_f = (-\infty, 1)$$

9.

$$2y^2 + x^2 + 1 = 0 \rightarrow \begin{cases} 2y_1^2 = -x_1^2 - 1 \\ 2y_2^2 = -x_2^2 - 1 \end{cases} \xrightarrow{x_1=x_2} x y_1^2 = x y_2^2 \rightarrow y_1^2 = y_2^2 \rightarrow y_1 = y_2 \rightarrow \text{تابع است}$$

$$x^2 + 4y^2 - 2x - 12y + 1 = 0 \rightarrow x^2 + 4y^2 - 2x - 12y + 9 + 1 = (x-1)^2 + (2y-3)^2$$

$$(x-1)^2 + (2y-3)^2 = 0 \rightarrow \begin{cases} x-1=0 \rightarrow x=1 \\ 2y-3=0 \rightarrow 2y=3 \rightarrow y=\frac{3}{2} \end{cases} \Rightarrow \begin{bmatrix} 1 \\ 3/2 \end{bmatrix} \text{ در این نقطه عبارت تابع است}$$

V.

$$x \sin y = y \sin x \xrightarrow{\lambda=0} 0 = y \sin 0 \rightarrow 0 = 0 \rightarrow \begin{cases} x=0 \\ y \in \mathbb{R} \end{cases}$$

ب. برای $x=0$ مقدار و ضابطه تابع نمی باشد

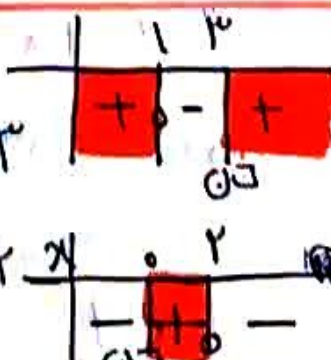
$$\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \xrightarrow{(\frac{x}{y})^2} \frac{x}{y} + \frac{y}{x} + 2\sqrt{\frac{xy}{yx}} \rightarrow \frac{x}{y} + \frac{y}{x} + 2 = 4 \rightarrow \frac{x}{y} + \frac{y}{x} = 2$$

$$\frac{x^2+y^2}{xy} = 2 \rightarrow x^2+y^2-2xy=0 \rightarrow (x-y)^2=0 \rightarrow x-y=0 \rightarrow x=y$$

بنابر ناصیه المربع (تابع)

Λ.

$$y = \sqrt{\frac{x-1}{x-3}} + \sqrt{\frac{3-x}{x}} \rightarrow \begin{cases} x-1=0 \rightarrow x=1 \\ x-3=0 \rightarrow x=3 \\ x=0 \\ 3-x=0 \rightarrow x=3 \end{cases}$$



$$(-\infty, 1] \cup (3, +\infty) \cap (0, 3] \rightarrow \mathbb{R} = (0, 3]$$

1. $\sqrt{x} + \sqrt{y-1} = \sqrt{3} \rightarrow \sqrt{y-1} = \sqrt{3} - \sqrt{x} \rightarrow y-1 = 3 + x - 4\sqrt{x}$
 $y = x + 4 - 2\sqrt{3x} \rightarrow 3x \geq 0 \rightarrow x \geq 0$

9. $y = \frac{\sqrt{x^2 - vx + 4}}{\sqrt{x - 6x + 14}}$ $\left. \begin{array}{l} x^2 - vx + 4 \geq 0 \rightarrow 1 + 4 = v \\ v - v = 0 \end{array} \right\} \begin{array}{l} x=1 \\ x = \frac{c}{a} = \frac{4}{1} = 4 \end{array}$
 جميع ریشه ها برابر است

x		1	4	
$x^2 - vx + 4$	+	-	+	+

 $\rightarrow (-\infty, 1] \cup [4, +\infty)$
 $\rightarrow -9x + 14 > 0 \rightarrow -9x > -14 \rightarrow x < \frac{14}{9} \rightarrow (-\infty, \frac{14}{9}]$
 $(-\infty, 1] \cup [4, +\infty) \cap (-\infty, \frac{14}{9}]$
 $(-\infty, 1]$

$\sqrt{\frac{x^2 - vx + 4}{x^2 - 10x + 14}} \rightarrow x^2 - vx + 4 \geq 0 \rightarrow \left\{ \begin{array}{l} x=1 \\ x = \frac{c}{a} = \frac{4}{1} = 4 \end{array} \right.$
 $x^2 - 10x + 14 \geq 0 \rightarrow (x-2)(x-8) \left\{ \begin{array}{l} x=2 \\ x=8 \end{array} \right.$

x		1	2	4	8	
$x^2 - vx + 4$	+	-	-	+	+	
$x^2 - 10x + 14$	+	+	-	-	+	

 $(-\infty, 1] \cup [2, 4] \cup [8, +\infty)$

10. $y = \frac{\sqrt{rx^2 - ax + r}}{\sqrt{rx^2 - vx + r}}$ $\rightarrow rx^2 - ax + r \geq 0 \rightarrow \left\{ \begin{array}{l} r + r = a \\ a - a = 0 \end{array} \right\} \begin{array}{l} x=1 \\ x = \frac{c}{a} = \frac{r}{r} \end{array}$
 جميع ریشه ها برابر است

x		1	$\frac{r}{r}$	
$rx^2 - ax + r$	+	-	+	

 $\rightarrow (-\infty, 1] \cup [\frac{r}{r}, +\infty)$
 $\rightarrow rx^2 - vx + r \geq 0 \rightarrow \left\{ \begin{array}{l} r + r = v \\ v - v = 0 \end{array} \right\} \begin{array}{l} x=1 \\ x = \frac{c}{a} = \frac{r}{r} \end{array}$

x		1	$\frac{r}{r}$	
$rx^2 - vx + r$	+	-	+	

 $\rightarrow (-\infty, 1] \cup [\frac{r}{r}, +\infty)$
 $(-\infty, 1] \cup [\frac{r}{r}, +\infty)$

$\sqrt{\frac{rx^2 - ax + r}{rx^2 - vx + r}} \rightarrow rx^2 - ax + r < 0 \rightarrow \left\{ \begin{array}{l} x=1 \\ x = \frac{r}{r} \end{array} \right.$
 $\rightarrow rx^2 - vx + r < 0 \rightarrow \left\{ \begin{array}{l} x=1 \\ x = \frac{r}{r} \end{array} \right.$

x		1	$\frac{r}{r}$	
$rx^2 - ax + r$	+	-	-	+
$rx^2 - vx + r$	+	-	+	+

 $(-\infty, 1) \cup (1, \frac{r}{r}) \cup [\frac{r}{r}, +\infty)$