

ايف) $ry^r + \alpha^r + 1 = 0 \Rightarrow \begin{cases} ry_1^r + 1 = -\alpha^r \\ ry_r^r + 1 = -\alpha^r \end{cases} \xrightarrow{\alpha_1 = \alpha^r} \begin{cases} ry_1^r = -\alpha^r - 1 \\ ry_r^r = -\alpha^r - 1 \end{cases}$
 $ry_1^r = ry_r^r \Rightarrow y_1^r = y_r^r = y^r$ يعني $y_1 = y_r$

ب) $x^p + cy^p - 1 = 0$ $\Rightarrow (x-1)^p + (cy-1)^p = 0$ (2)

$\Rightarrow x=1, y=\frac{1}{c}$ ✓

تابع $y = \sin x$ در نقطه $x=0$ مشتق می‌گیریم:

$$\frac{d}{dx} \sin x = \cos x$$

در $x=0$: $\cos 0 = 1$

بنابراین، $\left. \frac{dy}{dx} \right|_{x=0} = 1$

این نتیجه را می‌توان با استفاده از تعریف مشتق نیز بررسی کرد:

$$\lim_{h \rightarrow 0} \frac{\sin(0+h) - \sin(0)}{h} = \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$$

پس، $\left. \frac{dy}{dx} \right|_{x=0} = 1$

$$\text{c) } \left(\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} \right)^r = (r) = 0 \quad \frac{x}{y} + \frac{y}{x} + r \sqrt{\frac{xy}{yx}} = 1 \quad \underline{\underline{\sqrt{1}=1}} \Rightarrow$$

$$\frac{x}{y} + \frac{y}{x} + r = r \Rightarrow \frac{x}{y} + \frac{y}{x} = r \Rightarrow \frac{x^r + y^r}{x^r y^r} = r \Rightarrow x^r + y^r = r x^r y^r$$

$$\Rightarrow x^r + y^r - r x^r y^r = 0 \Rightarrow (x - y)^r = 0 \quad \left\{ \begin{array}{l} x=y \\ r \neq 0 \end{array} \right.$$

$$g \geq \sqrt{\frac{x-1}{x-1}} + \sqrt{\frac{1-x}{x}} \Rightarrow \frac{x-1}{x-1} \geq 0 \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$$

$$\frac{1-x}{x} \geq 0 \Rightarrow 1-x \geq 0 \Rightarrow x \leq 1$$

$$(0, 1] \cap ((-\infty, 1] \cup [1, +\infty)) = (0, 1] = D_f$$

$\sqrt{x} + \sqrt{y-1} = \sqrt{3}$
 $\hookrightarrow y-1 \geq 0 \Rightarrow y \geq 1$
 $(\sqrt{y-1}) = \sqrt{3-x} \xrightarrow{\sqrt{y-1} \geq 0} \sqrt{3-x} \geq 0 \wedge$
 $= \sqrt{3-x} \geq 0 \Rightarrow x \leq 3$
 $D_f: 0 \leq x \leq 3 \wedge [1, 3]$

$$\text{Gilt: } \begin{cases} \sqrt{x^2 - 10x + 14} > 0 \\ \sqrt{x^2 - 10x + 14} > 0 \end{cases} \Rightarrow \begin{cases} x^2 - 10x + 14 > 0 \\ x^2 - 10x + 14 > 0 \end{cases} \Rightarrow \begin{cases} (x-1)(x-9) > 0 \\ (x-1)(x-9) > 0 \end{cases} \Rightarrow \begin{cases} x < 1 \vee x > 9 \\ x < 1 \vee x > 9 \end{cases}$$

$\text{b) } \sqrt{\frac{x^2 - \sqrt{x} + 4}{x - 10x + 14}} \geq 0 \Rightarrow \frac{x^2 - \sqrt{x} + 4}{x^2 - 10x + 14} \geq 0 \rightarrow x^2 - \sqrt{x} + 4 \geq 0 \Rightarrow (x-1)(x-4) \geq 0 \Rightarrow \begin{matrix} x \geq 4 \\ x \leq 1 \end{matrix}$
 $\frac{1}{x-1} - \frac{1}{x-4} + \frac{1}{x-1} - \frac{1}{x-4} + \dots$

$\checkmark$ $\sqrt{4x^2 - 4x + 3} > 0 \Rightarrow \Delta = 4 - 12 = -8 \Rightarrow x = \frac{1 \pm i\sqrt{2}}{2} \Rightarrow x_1 = \frac{1+i\sqrt{2}}{2} \quad x_2 = \frac{1-i\sqrt{2}}{2}$
 $\checkmark$ $\sqrt{4x^2 - 4x + 5} > 0 \Rightarrow \Delta = 4 - 20 = -16 \Rightarrow x = \frac{1 \pm 2i}{2} \Rightarrow x_1 = \frac{1+2i}{2} \quad x_2 = \frac{1-2i}{2}$
 $D_f = (-\infty, 1) \cup [\frac{1}{2}, +\infty) \cap (-\frac{1}{2}, 1) \cup (\frac{1}{2}, +\infty) = (-\infty, 1) \cup (\frac{1}{2}, +\infty)$

$\sqrt{\frac{\gamma \alpha^2 - \alpha \kappa + \epsilon}{\gamma \alpha^2 - \gamma \alpha + \epsilon}} \geq 0 \rightarrow \gamma \alpha^2 - \alpha \kappa + \epsilon \geq 0 \Rightarrow \Delta \geq \kappa - \epsilon \geq 1$ $\alpha = \frac{\kappa+1}{\epsilon} \Rightarrow \alpha \geq \frac{\kappa}{\epsilon}$ $\alpha \geq 1$
 $\sqrt{\frac{\gamma \alpha^2 - \alpha \kappa + \epsilon}{\gamma \alpha^2 - \gamma \alpha + \epsilon}} \geq 0 \rightarrow \gamma \alpha^2 - \gamma \alpha + \epsilon \geq 0 \Rightarrow \Delta \leq \alpha - \epsilon \geq 1$ $\frac{1}{\frac{\kappa}{\epsilon}} + \frac{\frac{\kappa}{\epsilon}}{\frac{\kappa}{\epsilon}} = \frac{\epsilon}{\kappa} + \frac{\kappa}{\kappa} = \frac{\epsilon}{\kappa} + 1$
 $\Rightarrow \alpha \geq 1$ $\alpha = \frac{\kappa}{\epsilon}$ $(-\infty, 1) \cup (1, \frac{\kappa}{\epsilon}) \cup [\frac{\kappa}{\epsilon}, +\infty)$