

$A = \{1, 2, 3\}$ $A \times B - B^2 = ? \Rightarrow I - II = \{(1, 2), (1, 3)\}$ جواب

$B = \{2, 3\}$ $A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\} = I$
 $B \times B = \{(2, 2), (2, 3), (3, 2), (3, 3)\} = II$

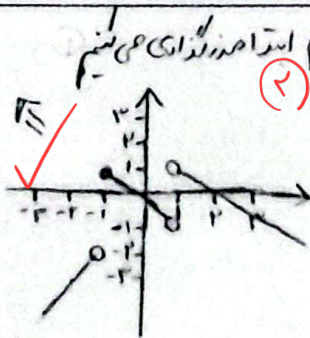
الف) $f(n) = \{(2, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 2)\}$
 به تابع بودن $\Rightarrow$ برای n های برابر y ها هم باید برابر باشند $\Rightarrow (2, m^2) = (2, m+2) \Rightarrow m^2 = m+2 \Rightarrow m^2 - m - 2 = 0$
 $\Rightarrow (m-2)(m+1) = 0 \Rightarrow m = 2, m = -1$
 اگر $m = 2 \Rightarrow (2, 4) \neq (2, 1)$ $\Rightarrow m = 2$ نادرست است $\Rightarrow m = -1$ صحیح است

ب) $g(n) = \{(3, m^2), (2, 1), (m, 2), (3, m+2), (-2, m), (-1, 3)\}$
 $(3, m^2) = (3, m+2) \Rightarrow m^2 = m+2 \Rightarrow m^2 - m - 2 = 0 \Rightarrow (m-2)(m+1) = 0$
 $\Rightarrow m = 2, m = -1$
 اگر $m = 2 \Rightarrow (2, 1) \neq (2, 4)$ $\Rightarrow m = 2$ نادرست است $\Rightarrow m = -1$ صحیح است

$b = -1, a + b = 0 \Rightarrow a = 1$
 $f(n) = \begin{cases} n^2 - 1 & ; n \leq 1 \\ an + b & ; 1 < n \leq 2 \\ n^3 & ; n \geq 2 \end{cases}$
 $a \times b = -1$ جواب $\Rightarrow f(1) = 1 - 1 = 0 = a + b$
 $f(n) = an + b = n^3 \Rightarrow f(2) = 2a + b = 8$
 $\Rightarrow \begin{cases} 2a + b = 8 \\ a + b = 0 \end{cases} \Rightarrow \begin{cases} a = 8 \\ b = -8 \end{cases}$

$f(n) = \begin{cases} 2n - 3 & ; n \geq k \\ 2 - 3n & ; n \leq k \end{cases}$
 $f(n) = 2n - 3 = 2 - 3n \Rightarrow n = k$
 $\Rightarrow f(k) = 2k - 3 = 2 - 3k \Rightarrow 5k = 4 \Rightarrow k = \frac{4}{5}$
 جواب $k = \frac{4}{5}$

الف) $f(n) = \begin{cases} -n + 2 & ; n > 1 \\ -n & ; -1 \leq n < 1 \\ n - 1 & ; n < -1 \end{cases}$
 $D_f = \mathbb{R} - \{1\}$
 $R_f = (-\infty, 1]$
 I) $n = 1 \Rightarrow f(1) = -1 + 2 = 1, f(2) = -2 + 2 = 0$
 II) $n = -1 \Rightarrow f(-1) = 1, n = 1 \Rightarrow f(1) = -1$
 III) $n = -1 \Rightarrow f(-1) = -1 - 1 = -2, f(-2) = -2 - 1 = -3$



$$D_f = (-\infty, 1) \cup \left(1, \frac{4}{3}\right) \cup \left[\frac{3}{2}, +\infty\right) \quad \text{ارائه قسمت ب سوال 10}$$

• الف) $2y^3 + x^2 + 1 = 0 \Rightarrow 2y^3 = -x^2 - 1$ باتوجه به تعریف تابع

$\rightarrow \begin{cases} 2y_1^3 = -x^2 - 1 \\ 2y_2^3 = -x^2 - 1 \end{cases} \rightarrow 2y_1^3 = 2y_2^3 \Rightarrow y_1^3 = y_2^3 \Rightarrow y_1 = y_2 \Rightarrow$ تابع یک به یک ✓

• ب) $x^2 + 2y^2 - 2x - 12y + 10 = 0 \Rightarrow \underbrace{x^2 - 2x + 1}_{(x-1)^2} + \underbrace{2y^2 - 12y + 9}_{(2y-3)^2} = 0$

$\Rightarrow (x-1)^2 + (2y-3)^2 = 0 \Rightarrow \begin{cases} x=1 \\ y=\frac{3}{2} \end{cases} \Rightarrow \left(\frac{1}{2}, \frac{3}{2}\right)$ ✓ تابع یک به یک

• الف) $x \sin y = y \sin x \xrightarrow{\text{موانع نقطه}} x=0 \Rightarrow (0) \sin y = y \sin(0) \Rightarrow y \in \mathbb{R}$

✓ تابع نسیب ✓

• ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \xrightarrow{\text{دو طرفه توان 2}} \frac{x}{y} + \frac{y}{x} + 2\sqrt{\frac{1}{xy}} = 4 \Rightarrow \frac{x}{y} + \frac{y}{x} + 2 = 4$

$\Rightarrow \frac{x}{y} + \frac{y}{x} = 2 \Rightarrow \frac{x^2 + y^2}{xy} = 2 \Rightarrow x^2 + y^2 = 2xy \Rightarrow x^2 - 2xy + y^2 = 0 \Rightarrow (x-y)^2 = 0$

$\Rightarrow x=y$ ✓ تابع است ✓

• الف) $y = \sqrt{\frac{n-1}{n-3}} + \sqrt{\frac{2-n}{n}}$

$\frac{n-1}{n-3} \geq 0 \Rightarrow \begin{cases} n=1 \\ n=3 \end{cases}$ ✓

$\frac{2-n}{n} \geq 0 \Rightarrow \begin{cases} n=0 \\ n=2 \end{cases}$ ✓

$(-\infty, 1] \cup (3, +\infty) \cap (0, 2] = (0, 1] = D_f$ ✓

• ب) $\sqrt{n} + \sqrt{y-1} = \sqrt{3}$

$\sqrt{y-1} = \sqrt{3} - \sqrt{n}$

$n > 0, \sqrt{y-1} \geq 0 \Rightarrow \sqrt{3} - \sqrt{n} \geq 0 \Rightarrow \sqrt{n} \leq \sqrt{3} \Rightarrow n \leq 3$

$\Rightarrow D_f = [0, 3]$ ✓

• الف) $y = \frac{\sqrt{n^2 - 7n + 4}}{\sqrt{n - 10n + 14}}$ ✓

$n^2 - 7n + 4 = (n-1)(n-4) \geq 0 \Rightarrow \begin{cases} n=1 \\ n=4 \end{cases}$ ✓

$n - 10n + 14 = -9n + 14 \geq 0 \Rightarrow n = \frac{14}{9}$

$(-\infty, 1] \cup [4, +\infty) \cap (-\infty, \frac{14}{9}] = (-\infty, 1] = D_f$ ✓

• ب) $y = \sqrt{\frac{n^2 - 7n + 4}{n^2 - 10n + 14}}$ ✓

$\frac{n^2 - 7n + 4}{n^2 - 10n + 14} \geq 0$

$n^2 - 7n + 4 = (n-1)(n-4) \Rightarrow \begin{cases} n=1 \\ n=4 \end{cases}$

$n^2 - 10n + 14 = (n-2)(n-7) \Rightarrow \begin{cases} n=2 \\ n=7 \end{cases}$

$D_f = (-\infty, 1] \cup (2, 4] \cup (7, +\infty)$ ✓

• الف) $y = \frac{\sqrt{2n^2 - 5n + 3}}{\sqrt{3n^2 - 7n + 4}}$ ✓

$2n^2 - 5n + 3 \geq 0 \Rightarrow \Delta = 25 - 4(2)(3) = 1 \Rightarrow n = \frac{5 \pm 1}{4} = \frac{3}{2}, 1$

$3n^2 - 7n + 4 \geq 0 \Rightarrow \Delta = 49 - 4(3)(4) = -23 < 0 \Rightarrow$ همیشه مثبت

$n = \frac{5+1}{4} = \frac{3}{2}, n = \frac{5-1}{4} = 1$

$(-\infty, 1] \cup [\frac{3}{2}, +\infty) = I$ ✓

• ب) $y = \sqrt{\frac{2n^2 - 5n + 3}{3n^2 - 7n + 4}}$ ✓

$\frac{2n^2 - 5n + 3}{3n^2 - 7n + 4} \geq 0$

$2n^2 - 5n + 3 = (2n-3)(n-1) \Rightarrow \begin{cases} n=\frac{3}{2} \\ n=1 \end{cases}$

$3n^2 - 7n + 4$ همیشه مثبت

$(-\infty, 1] \cup [\frac{3}{2}, +\infty) = II$ ✓

$I \cap II - \left\{\frac{3}{2}, 1\right\} = (-\infty, 1) \cup \left[\frac{3}{2}, +\infty\right) = D_f$ ✓

$n \neq \frac{3}{2}, 1$

رابطه دایره $a^2 + b^2 + c^2 = 0 \Rightarrow \begin{cases} a=0 \\ b=0 \\ c=0 \end{cases} \Rightarrow \begin{cases} m=1 \\ m=\frac{1}{2} \end{cases}$

