

$$A \times B - B^2 =$$

$$\{(1,2), (2,2), (3,2), (1,3), (2,3), (3,3)\} - \{(2,2), (2,3), (3,2), (3,3)\} = \{(1,2), (1,3)\}$$

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الف) $f(x) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$

$$m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow m = \frac{-1 \pm \sqrt{1+8}}{2} \rightarrow m = 1$$

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ب) $g(x) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (-2, m), (-1, 3)\}$

$m^2 = m+2 \rightarrow m = \frac{-1 \pm \sqrt{1+8}}{2}$ هر دو ریشه صحیح هستند

$$f(x) = \begin{cases} x^2 - 1 & ; x \leq 1 \\ ax + b & ; 1 \leq x \leq 2 \\ x^3 & ; x \geq 2 \end{cases}$$

$$x=1 \rightarrow \begin{cases} a+b = 1^2 - 1 \\ a+b = 0 \end{cases}$$

$$x=2 \rightarrow \begin{cases} 2a+b = 2^2 \\ 2a+b = 8 \end{cases}$$

$$\begin{cases} a+b=0 \\ 2a+b=8 \end{cases} \rightarrow a=8, b=-8$$

$$ax+b = 8x-8 = -4x$$

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$$f(x) = \begin{cases} 2x-3 & ; x \geq k \\ 4-3x & ; x \leq k \end{cases}$$

$$2x-3 = 4-3x$$

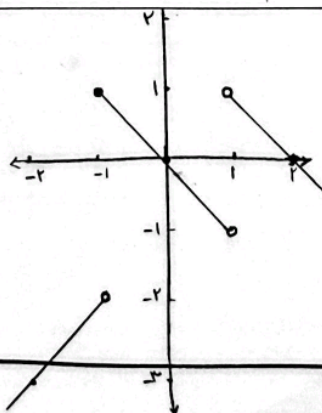
$$5x = 7$$

$$x = \frac{7}{5} = 1.4 = k$$

هنگامی که $x=k$ است مقدار دو ضابطه نیز برابر است

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$$f(x) = \begin{cases} -x+2 & ; x > 1 \\ -x & ; -1 \leq x \leq 1 \\ x-1 & ; x < -1 \end{cases}$$



$$D_f = \mathbb{R} - \{1\}$$

$$R_f = (-\infty, -2) \cup (-1, 1]$$

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الف) $xy^r + x^r + 1 = 0$

$\begin{cases} y_1^r = -x^r - 1 \\ xy_1^r = -x^r - 1 \end{cases} \rightarrow xy_1^r = y_1^r \rightarrow y_1^r = y_1^r \rightarrow y_1 = y_1 \checkmark$

ب) $x^r + 4y^r - 12y + 1 = 0 \rightarrow (x-1)^r + (y-3)^r = 0 \rightarrow x=1, y=3 \checkmark$ نقطه $(1, 3)$ تابع هست

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الف) $x \sin y = y \sin x \propto$ تابع نیست

$x=0 \rightarrow 0 \sin y = y \sin(0) \rightarrow 0=0 \rightarrow y \in \mathbb{R}$

ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \rightarrow x=y$ تابع نیست

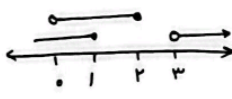
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الف) $y = \sqrt{\frac{x-1}{x-3}} + \sqrt{\frac{2-x}{x}}$

$\frac{x-1}{x-3} \geq 0 \quad \begin{array}{c|c|c|c|} 1 & 3 & & \\ \hline + & - & + & + \end{array} \textcircled{1}$

$\frac{2-x}{x} \geq 0 \quad \begin{array}{c|c|c|c|} 2 & 0 & & \\ \hline - & + & - & - \end{array} \textcircled{2}$

$\textcircled{1} \cap \textcircled{2} = (0, 1]$



ب) $\sqrt{x} + \sqrt{y-1} = \sqrt{3}$

$\sqrt{y-1} = \sqrt{3} - \sqrt{x}$

$\sqrt{3} - \sqrt{x} \geq 0$

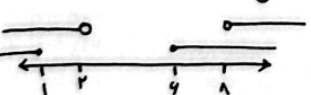
$\sqrt{x} \leq \sqrt{3} \rightarrow 0 \leq x \leq 3 = D_f$

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الف) $y = \frac{\sqrt{x^2 - 5x + 4}}{\sqrt{x^2 - 10x + 14}}$

$x^2 - 5x + 4 \geq 0 \quad \begin{array}{c|c|c|c|} 1 & 4 & & \\ \hline + & - & + & + \end{array} \textcircled{1}$

$x^2 - 10x + 14 \geq 0 \quad \begin{array}{c|c|c|c|} 2 & 7 & & \\ \hline + & - & + & + \end{array} \textcircled{2}$



$D_f = \textcircled{1} \cap \textcircled{2} = (-\infty, 1] \cup (4, 7)$

ب) $y = \sqrt{\frac{x^2 - 5x + 4}{x^2 - 10x + 14}}$

$\frac{x^2 - 5x + 4}{x^2 - 10x + 14} \geq 0$

$\begin{array}{c|c|c|c|c|c|} 1 & 2 & 4 & 7 & & \\ \hline + & - & + & - & + & + \end{array}$

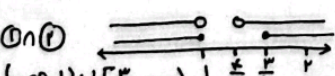
$D_f = (-\infty, 1] \cup (2, 4] \cup (7, +\infty)$

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الف) $y = \frac{\sqrt{2x^2 - 5x + 3}}{\sqrt{3x^2 - 7x + 4}}$

$2x^2 - 5x + 3 \geq 0 \quad \begin{array}{c|c|c|c|} 1 & 3/2 & & \\ \hline + & - & + & + \end{array} \textcircled{1}$

$3x^2 - 7x + 4 \geq 0 \quad \begin{array}{c|c|c|c|} 1 & 4/3 & & \\ \hline + & - & + & + \end{array} \textcircled{2}$



$D_f = (-\infty, 1] \cup [4/3, 2)$

ب) $y = \sqrt{\frac{2x^2 - 5x + 3}{3x^2 - 7x + 4}}$

$\frac{2x^2 - 5x + 3}{3x^2 - 7x + 4} \geq 0 \quad \begin{array}{c|c|c|c|c|c|} 1 & 3/2 & 4/3 & 2 & & \\ \hline + & - & + & - & + & + \end{array}$

$D_f = (-\infty, 1] \cup (1, 4/3) \cup [4/3, 2)$

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