

$$A \times B - B^2 =$$

$$\{(1,2), (2,2), (3,2), (1,3), (2,3), (3,3)\} - \{(2,2), (2,3), (3,2), (3,3)\} = \{(1,2), (1,3)\}$$

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$$\text{الف) } f(x) = \{(2, m^2), (2, 1), (-3, m), (-2, m), (2, m+2), (m, 4)\}$$

$$m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow m = \frac{-1 \pm \sqrt{1+20}}{2} \rightarrow m = 1$$

$$\text{ب) } g(x) = \{(2, m^2), (2, 1), (m, 4), (2, m+2), (-2, m), (-1, 3)\}$$

$$m^2 = m+2 \rightarrow m = \frac{-1 \pm \sqrt{1+20}}{2} \rightarrow \text{هر دو ریشه صحیح نیستند} \rightarrow \text{به ازای هیچ مقدار حقیقی } m \text{ تابع نیست}$$

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$$f(x) = \begin{cases} x^2 - 1 & ; x \leq 1 \\ ax + b & ; 1 \leq x \leq 2 \\ x^3 & ; x \geq 2 \end{cases}$$

$$x=1 \rightarrow \begin{cases} a+b = x^2 - 1 \\ a+b = 0 \end{cases}$$

$$x=2 \rightarrow \begin{cases} 2a+b = x^3 \\ 2a+b = 8 \end{cases}$$

$$\begin{cases} a+b=0 \\ 2a+b=8 \end{cases} \rightarrow a=8, b=-8$$

$$ax+b = 8x-8 = -4x$$

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$$f(x) = \begin{cases} 2x-3 & ; x \geq k \\ 4-3x & ; x \leq k \end{cases}$$

$$2x-3 = 4-3x$$

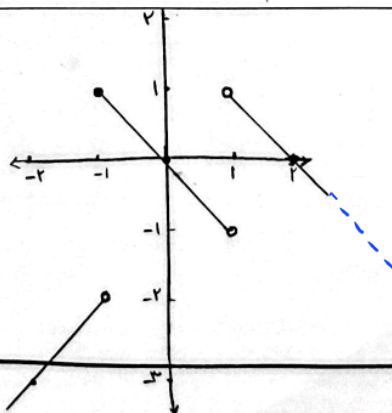
$$5x = 7$$

$$x = \frac{7}{5} = 1.4 = k$$

هنگامی که $x=k$ است مقدار دو ضابطه نیز برابر است

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$$f(x) = \begin{cases} -x+2 & ; x > 1 \\ -x & ; -1 \leq x \leq 1 \\ x-1 & ; x < -1 \end{cases}$$



$$D_f = \mathbb{R} - \{1\}$$

$$R_f = (-\infty, -1) \cup (-1, 1]$$

$$R_f = (-\infty, 1]$$

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الف) $xy^2 + x^2 + 1 = 0$

$\begin{cases} xy_1^2 = x^2 - 1 \\ xy_2^2 = -x^2 - 1 \end{cases} \rightarrow xy_1^2 = xy_2^2 \rightarrow y_1^2 = y_2^2 \rightarrow y_1 = y_2 \checkmark$



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ب) $x^2 + xy^2 - 2x - 12y + 14 = 0 \rightarrow (x-1)^2 + (y-3)^2 = 0 \rightarrow x=1, y=3$ تنها نقطه $\sqrt{(1,3)}$ تابع هست

الف) $x \sin y = y \sin x \propto$ تابع نیست

$x=0 \rightarrow 0 \sin y = y \sin 0 \rightarrow 0=0 \rightarrow y \in \mathbb{R}$



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ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \rightarrow x=y$ تابع است $\checkmark$

الف) $y = \sqrt{\frac{x-1}{x-3}} + \sqrt{\frac{2-x}{x}}$

$\frac{x-1}{x-3} \geq 0 \rightarrow \frac{1}{+} \frac{1}{-} \frac{3}{+} \textcircled{1}$

$\frac{2-x}{x} \geq 0 \rightarrow \frac{2}{+} \frac{1}{-} \frac{0}{+} \textcircled{2}$

$\textcircled{1} \cap \textcircled{2} = (0, 1]$



ب) $\sqrt{x} + \sqrt{y-1} = \sqrt{3}$

$\sqrt{y-1} = \sqrt{3} - \sqrt{x} \checkmark$

$\sqrt{3} - \sqrt{x} \geq 0$

$\sqrt{x} \leq \sqrt{3} \rightarrow 0 \leq x \leq 3 = D_f$

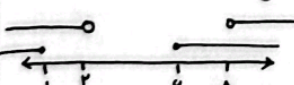


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الف) $y = \frac{\sqrt{x^2 - 5x + 4}}{\sqrt{x^2 - 10x + 14}}$

$x^2 - 5x + 4 \geq 0 \rightarrow \frac{1}{+} \frac{4}{-} \frac{1}{+} \textcircled{1}$

$x^2 - 10x + 14 \geq 0 \rightarrow \frac{2}{+} \frac{7}{-} \frac{14}{+} \textcircled{2}$



$D_f = \textcircled{1} \cap \textcircled{2} = (-\infty, 1] \cup (4, +\infty)$

$-4m+14 > 0$
 $m < \frac{14}{4}$
 $m < 1$

ب) $y = \sqrt{\frac{x^2 - 5x + 4}{x^2 - 10x + 14}}$

$\frac{x^2 - 5x + 4}{x^2 - 10x + 14} \geq 0$

$\frac{1}{+} \frac{4}{-} \frac{1}{+} \frac{2}{+} \frac{7}{-} \frac{14}{+}$

$D_f = (-\infty, 1] \cup (2, 4] \cup (14, +\infty)$

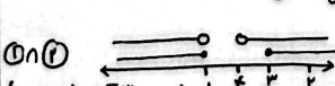
$(1, 0)$

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الف) $y = \frac{\sqrt{2x^2 - 5x + 3}}{\sqrt{3x^2 - 7x + 4}}$

$2x^2 - 5x + 3 \geq 0 \rightarrow \frac{1}{+} \frac{3}{-} \frac{1}{+} \textcircled{1}$

$3x^2 - 7x + 4 \geq 0 \rightarrow \frac{1}{+} \frac{4}{-} \frac{7}{+} \frac{1}{+} \textcircled{2} \checkmark$



$D_f = (-\infty, 1] \cup [\frac{3}{2}, +\infty)$
 $\hookrightarrow [\frac{3}{2}, +\infty)$

ب) $y = \sqrt{\frac{2x^2 - 5x + 3}{3x^2 - 7x + 4}}$

$\frac{2x^2 - 5x + 3}{3x^2 - 7x + 4} \geq 0$

$\frac{1}{+} \frac{3}{-} \frac{1}{+} \frac{4}{+} \frac{7}{-} \frac{1}{+}$

$D_f = (-\infty, 1) \cup (1, \frac{3}{2}) \cup [\frac{3}{2}, +\infty)$



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