

Subject: ()

Date: هفتی نصیری نژاد

سوال ۸۱

$$f(x) = \begin{cases} (a+1)(x+2) & : x > 1 \\ 3a+2x & : x \leq 1 \end{cases}$$

$$2f(a) = f(-2) + a$$

$a = -1, 1$

برای $x = a$ و $x = -2$

$$\begin{cases} (a+2)(a+1) \\ 3a-4 \end{cases} \xrightarrow{\text{مقایسه}} 2x \vee x(a+1) = 3a-4+a$$

$$14a+14 = 4a-4 \rightarrow 10a = -18 \rightarrow a = -1, 1$$

سوال ۸۲

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = ?$$

مقرّر می‌کنیم

$$(2-\sqrt{3})(2+\sqrt{3}) = 1$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2$$

$$= 2\sqrt{2-\sqrt{3}} + 2\sqrt{2+\sqrt{3}} + 4$$

$$\xrightarrow{\text{توان ۲}} (\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}})^2 = 2-\sqrt{3} + 2+\sqrt{3} + 2 = 4 \xrightarrow{\text{جذر}} \sqrt{4}$$

$$\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} = \sqrt{4} \rightarrow 2(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}) + 4 = 2\sqrt{4} + 4$$

سوال ۸۳

$$x = -x \rightarrow 2f(-x) - 3f(x) = 4x^2 + x \xrightarrow{\times 2} 4f(-x) - 6f(x) = 8x^2 + 2x$$

$$x = x \rightarrow 2f(x) - 3f(-x) = 8x^2 - x \xrightarrow{\times 2} 4f(x) - 6f(-x) = 16x^2 - 2x$$

$$-2f(x) = 12x^2 + x$$

$$f(x) = -\frac{6x^2}{2} - \frac{x}{2}$$

$$2f(x) - 3f(-x) = 8x^2 - x$$

سوال ۸۴

$$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m + 1$$

① $x=0 \rightarrow 2f(0) = 3m-1 \times 3 \rightarrow 4f(0) = 9m-3$

$$x = -\frac{2}{3} \rightarrow 4f(0) = 14 + 2m + 3m + 1 \rightarrow 4m = 14 \rightarrow m = \frac{7}{2} = 3, 5$$

① $\rightarrow 2f(0) = 3(\frac{7}{2}) - 1 \rightarrow f(0) = \frac{17}{4}$

— Arman —

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \rightarrow 3x - 12 + \frac{3}{x} \xrightarrow{ax+b} \text{ سوال 85}$$

$$\frac{3}{a}\left(x + \frac{1}{x}\right) - \frac{12}{b} \rightarrow a=3, \frac{1}{b}=-12 \rightarrow b=-\frac{1}{12}$$

$$f(x) = ax + b \rightarrow f(-1) = -3 - \frac{1}{12} = -\frac{37}{12}$$

$$\begin{cases} f(x) = (x-1)\sqrt{1-x} \rightarrow x-1 = -(1-x) \rightarrow f(x) = -(1-x)^{\frac{3}{2}} \\ g(x) = -\sqrt{(1-x)^3} \rightarrow -(1-x)^{\frac{3}{2}} \end{cases} \rightarrow x \leq 1$$

$$g(x) = f(x) \quad \text{برابرند}$$

$$\begin{cases} f(x) = x\sqrt{x} \rightarrow \sqrt{x^2 \cdot x} = \sqrt{x^3} \rightarrow x \geq 0 \rightarrow \sqrt{x^3} = x \\ g(x) = \sqrt{x^3} \rightarrow x \geq 0 \end{cases} \quad f(x) = g(x) \quad \text{برابرند}$$

$$\begin{cases} f(x) = [\sqrt{x} - [\sqrt{x}]] \rightarrow D_f = [0, +\infty) \leftarrow x \geq 0 \\ g(x) = \sqrt{x - [x]} \rightarrow D_g \xrightarrow{x \geq 0} \sqrt{x - x} = 0 \end{cases} \quad \text{برابرند}$$

$$f(x) = [\sqrt{x} - [\sqrt{x}]] \xrightarrow{0 \leq t - [t]} \text{ این فرجه هر عددی که بین صفر و 1 باشد، برابر صفر است. بنابراین برای تمام } x \geq 0 \text{ داریم } f(x) = 0$$

$$\text{هر تابع دارای دامنه یکسان} \quad [0, +\infty) = D_f = D_g$$

$$f(x) = g(x) \quad \text{برابرند} \quad f(x) = g(x) = 0$$

$$f = \{ (1, 2), (2, 3), (3, 1), (0, 0) \}$$

سوال 89

$$g = \{ (-1, 1), (1, 1), (2, 3), (3, 5), (0, 2), (\sqrt{5}, 1) \}$$

$$\text{الف) } \left(\frac{f+g}{f-g} \right) (1) = \frac{2+1}{2-1} = 3$$

$$\text{ب) } \left(\frac{f+g}{f-g} \right) (2) = \frac{3+3}{3-3} = \frac{6}{0} \quad \text{تعريف نسبه}$$

$$\text{ج) } f(0)g(-1) = 0 \times 1 = 0$$

$$f(x) = \{ (2, 1), (-1, 4), (0, 5), (-2, 2), (3, -1) \}$$

سوال 810

$$\rightarrow 3x-1=2 \rightarrow x=1, 3x-1=-1 \rightarrow x=0$$

$$\text{الف) } g(x) = f(3x-1) \quad g(x) = \{ (1, 1), (0, 4), \left(\frac{1}{3}, 5\right), \left(-\frac{1}{3}, 2\right), \left(\frac{2}{3}, -1\right) \}$$

$$\text{ب) } h(x) = f(|x|) \rightarrow h(x) = \{ (2, 1), (-2, 1), (0, 5), (3, -1), (-3, -1) \} \checkmark$$