

1

$$f(x) = \sqrt{(a-2)x^2 + ax + b}$$

فقط در همین حالت یک عبارت درجه ۲ یک ریشه دارد و اینکه

$(x-3)^2$ باشد اما این ممکن نیست زیرا زیر رادیکال توان ۲ هواره تعریف شده است

هواره برقرار $(x-3)^2$

$a-2=0 \rightarrow a=2$
 $2x+b=0 \rightarrow b=-4$

2

$$f(x) = \sqrt{x^2 + 2x + 2 - m}$$

$D_f = \mathbb{R} \rightarrow x^2 + 2x + 2 - m \geq 0$

در این سوال $a > 0$ هواره

$-\frac{b}{2a} = -1$

$1 - (-1) = 2$

$\Delta \leq 0 \rightarrow 4 - 4(2 - m) \leq 0 \rightarrow 4 - 8 + 4m \leq 0 \rightarrow 4m \leq 4 \rightarrow m \leq 1$

3

$$f(x) = \frac{\sqrt{4-x^2}}{[x] + [-x] + 1}$$

$4-x^2 \geq 0 \rightarrow x^2 \leq 4 \rightarrow -2 \leq x \leq 2$

$[x] + [-x] + 1 \neq 0$

هواره معبر است
مکرر اعداد صحیح

$D_f = \{k | k \in \mathbb{Z}\}$

$*_1, *_2 = -2, -1, 0, 1, 2 = D_f$

4

$$y = \sqrt{x^2 + 3x^2} \rightarrow x^2 + 3x^2 \geq 0 \rightarrow x^2(1+3) \geq 0 \rightarrow x^2 \geq 0$$

$x=0 \rightarrow x=-2$

$D_f = [-2, +\infty)$

5

ب) $\sqrt{\frac{(x^2-1)(x^3-3x^2)}{x^3-1}}$

$x^2(x-1) = 0 \rightarrow x=0, x=1$

$\frac{(x^2-1)(x^3-3x^2)}{x^3-1} \geq 0$

$\frac{(x-1)(x^2+1-x)}{x^3-1} \geq 0$

$D_f = \{0\} \cup (-\infty, -1] \cup [2, +\infty)$

6

الف) $y = \sqrt{|x-2| - 3}$

$|x-2| \geq 3 \rightarrow x-2 \geq 3 \rightarrow x \geq 5$
 $x-2 \leq -3 \rightarrow x \leq -1$

$D_f = (-\infty, -1] \cup [5, +\infty)$

7

ب) $y = \sqrt{4-|x-2|}$

$4-|x-2| \geq 0 \rightarrow -4 \leq x-2 \leq 4 \rightarrow -2 \leq x \leq 6$

$D_f = [-2, 6]$

الف) $y = \sqrt{1 - x|x|}$ $D_f = [-1, 1] \cup (-\infty, 0] = (-\infty, 1]$

$x < 0 \rightarrow -x^2 \leq 1 \rightarrow x^2 \geq -1 \rightarrow$ همیشه برقرار است $\rightarrow x < 0$ در دامنه است!

$x \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1 \rightarrow$ $D_f = [-1, 1]$

از هر دو رابطه داریم $\rightarrow D_f = (-\infty, 1]$

ب) $y = \sqrt{3 - |x - 2| - 1}$ $D_f = (2, 5)$

صورت سوال را اشتباه نرست! $\rightarrow 3 - |x - 2| - 1 \geq 0 \rightarrow 2 - |x - 2| \geq 0 \rightarrow |x - 2| \leq 2$

$|x - 2| \leq 2 \rightarrow x - 2 \leq 2 \rightarrow x \leq 4$
 $|x - 2| \leq 2 \rightarrow -x + 2 \leq 2 \rightarrow x \geq 0$

$\rightarrow D_f = [0, 4]$

الف) $y = \frac{\sin^2 x - 1}{\cos x}$ $\rightarrow \cos x \neq 0 \rightarrow \bigoplus \rightarrow D_f = \mathbb{R} - \{k\pi + \frac{\pi}{2}\}$

ب) $y = \frac{\cos x}{x \cos x - 1}$ $\rightarrow \cos x \neq \frac{1}{x} \rightarrow \bigoplus \rightarrow D_f = \mathbb{R} - \{k\pi + \frac{\pi}{x}, k\pi - \frac{\pi}{x}\}$

الف) $\frac{x+3}{|2x+1| - |x+2|}$ $D_f = \mathbb{R} - \{-\frac{5}{2}, \frac{1}{2}\}$

$|2x+1| = |x+2| \rightarrow \pm(2x+1) = \pm(x+2) \rightarrow 2x+1 = x+2 \rightarrow x=1$
 $2x+1 = -(x+2) \rightarrow 2x+1 = -x-2 \rightarrow 3x = -3 \rightarrow x = -1$

$\rightarrow D_f = \mathbb{R} - \{1, -1\}$

ب) $|2x+1| - |x+2| \geq 0 \rightarrow (x-2)(x+4) \geq 0 \rightarrow x \leq -4 \text{ or } x \geq 2$

$D_f = (-\infty, -4] \cup [2, \infty)$

$2x-1 > 0 \rightarrow x > \frac{1}{2}$
 $\log_a^{2x-1} > 0 \rightarrow 2x-1 > 1 \rightarrow x > 1$
 $\log_{\frac{1}{10}}^{2x-1} \geq 0 \rightarrow \log_a^{2x-1} \leq 1 \rightarrow 2x-1 \leq a \rightarrow x \leq \frac{a+1}{2}$

$\rightarrow D_f = (\frac{1}{2}, \frac{a+1}{2}]$

$x \cos x + 1 > 0 \rightarrow \cos x > -\frac{1}{x}$ $D_f = [k\pi + \frac{\pi}{2}, k\pi + \frac{3\pi}{2}] \cup (k\pi - \frac{\pi}{2}, k\pi + \frac{\pi}{2})$

$\log \frac{x-1}{x+1} \geq 0 \rightarrow \frac{x-1}{x+1} \geq 1 \rightarrow x-1 \geq x+1 \rightarrow -2 \geq 0$ (غیرممکن)

$\log \frac{x-1}{x+1} \geq 0 \rightarrow \frac{x-1}{x+1} \leq 0 \rightarrow x-1 \leq 0 \text{ and } x+1 > 0 \rightarrow x \leq 1 \text{ and } x > -1 \rightarrow x \in (-1, 1]$

$D_f = (-1, 1]$

$$x \geq 0 \rightarrow 1 - x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1 \xrightarrow{x \geq 0} 0 \leq x \leq 1$$

$$x \leq 0 \rightarrow 1 + x^2 \geq 0$$

(الف - 4) $1 - x|x| \geq 0$

$D_f = (-\infty, 1]$

همواره مثبت $\rightarrow$

البته مرتعاشیم مثل تابع را رسم کنیم

(ب - 4) $3 - |x-5| \geq 0 \rightarrow |x-5| \leq 3 \rightarrow -3 \leq x-5 \leq 3 \rightarrow 2 \leq x \leq 8$

$\sqrt{3 - |x-5|} - 1 \neq 0 \rightarrow \sqrt{3 - |x-5|} \neq 1 \rightarrow 3 - |x-5| \neq 2 \rightarrow |x-5| \neq 1 \rightarrow x \neq 4, 6$

$D_f = [2, 8] - \{4, 6\} = [2, 4) \cup (4, 6) \cup (6, 8]$

(ب - 10) $\frac{x-1}{x+1} > 0 \rightarrow \boxed{x > 1 \text{ و } x < -1}$

$\log_{\frac{x+1}{x-1}} \geq 0 \rightarrow \frac{x-1}{x+1} \geq (1) \rightarrow \frac{x-1}{x+1} \geq 1 \rightarrow \frac{x-1}{x+1} - 1 \geq 0 \rightarrow \frac{x-1-x-1}{x+1} \geq 0 \rightarrow \frac{-2}{x+1} \geq 0$

$x+1 < 0 \rightarrow \boxed{x < -1} \rightarrow D_f = (-\infty, -1)$