آرزو شا یاوری — یازدهم دفتر ۸ — تکلیف شماره ۹

۱- $f(x)=\begin{cases}\cot\frac{\pi x}{4} & ; x\le 1\\ \sqrt{x^2+1} & ; x>1\end{cases}$ $\qquad \cot\frac{\pi}{6}=\sqrt{3} \longrightarrow \sqrt{(\sqrt{3})^2+1}=\sqrt{4}=2$

$\Rightarrow fof(\frac{2}{3})=2$

۲- الف) $2\cos^2(\frac{\pi}{3})=2(\frac{1}{2})^2=\frac{1}{2} \longrightarrow \sqrt{\frac{2(2)-1}{(2)^2}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

$fog(\frac{\pi}{3})=\frac{\sqrt{3}}{2}$

ب) $\frac{\sqrt{2}}{1-\sqrt{2}}=\frac{\sqrt{2}+2}{1-2}=-\sqrt{2}-2 \longrightarrow [-\sqrt{2}-2]=-4 \qquad fog(\sqrt{2})=-4$

۳- $\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2} \longrightarrow \frac{\sqrt{2}}{2}\sqrt{1-(\frac{\sqrt{2}}{2})^2}=\frac{1}{2} \qquad gof(\frac{\pi}{4})=\frac{1}{2}$

۴- الف) $fog(x)=\{(2,5)(4,5)(6,12)(8,12)\}$

ب) $gof(x)=\{\ \}$

ج) $fof(x)=\{\ \}$

د) $gog(x)=\{(2,6)(4,8)(6,10)\}$

۵- $(4,2)\in fog \qquad (a,3)\to(3,2)\to a=4 \qquad (a,b)=(4,5)$

$(4,1)\in gof \qquad (4,5)\to(b,1)\to b=5$

۶- $f(f(x))=a(ax+b)+b=a^2x+ab+b=4x+3 \qquad a^2=4\to a=\pm 2$

$a=+2\to 3b=3\to b=1 \to f(x)=2x+1 \xrightarrow{x=-1} 2(-1)+1=-1$

$a=-2\to -b=3\to b=-3 \to f(x)=-2x-3 \xrightarrow{x=-1} -2(-1)-3=-1 \qquad \Rightarrow f(-1)=-1$

$2x+3=-1\to 2x=-4\to x=-2 \to g(-1)=3(-2)-2=-8 \qquad gof(-1)=-8$

۷- $D_{gof}=\{x\in D_f ; f(x)\in D_g\} \qquad D_f=x+|x|\ge 0=\mathbb{R} \qquad D_g=\underbrace{x^2-4x}_{x(x-4)}\neq 0\to x\neq 0\ x\neq 4$

$\{x\in\mathbb{R} : \sqrt{x+|x|}\in\mathbb{R}-\{0,4\}\} \longrightarrow D_{gof}=(0,+\infty)-\{8\}$

s.a.m

$D_{(f+g)\circ f} = \{x \in D_f : f(x) \in D_{f+g}\}$ $\quad D_f = 1-x^2 \geqslant 0 \to 1 \geqslant x^2 \to -1 \leqslant x \leqslant 1$ ۸

$D_{f+g} : \sqrt{x} + \sqrt{1-x^2} \to x \geqslant 0 \cap -1 \leqslant x \leqslant 1 \to D_{f+g} = [0,1]$

$D_{(f+g)\circ f} = \{x \in [-1,1] : x \in [-1,1]\} \Rightarrow D_{(f+g)\circ f} = [-1,1]$

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5 \to \frac{3x+1}{x-2} = t \to t(x-2) = 3x+1 \to tx - 2t = 3x+1$ ۹

$\to 3x - tx = -2t - 1 \to x(t-3) = 2t+1 \to x = \frac{2t+1}{t-3}$

$\to 4x+5 = 4\left(\frac{2t+1}{t-3}\right) + 5 \to 4x+5 = \frac{8t+4+5(t-3)}{t-3} = \frac{13t-11}{t-3} \Rightarrow f(t) = \frac{13t-11}{t-3}$

$f(x) = \frac{13x-11}{x-3} = \frac{8x+4}{x-3} + 5$

ب) $f\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3}$ $\quad x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$\Rightarrow f(x) = x^3 - 3x$

$g(1) = 0 \quad g(2\sqrt{2}) = 0 \quad gof(x) = g(f(x)) \Rightarrow f(x) = 1 \ , \ f(x) = 2\sqrt{2}$ ۱۰

$\to x\sqrt{x} = 1 \to x = 1 \quad x\sqrt{x} = 2\sqrt{2} \to x = 2 \quad gof = \{(1,0)(2,0)\}$

$2 - 1 = 1$