

$$f\left(\frac{\sqrt{x}}{x}\right) = \cot \frac{\pi \frac{\sqrt{x}}{x}}{\frac{x}{x}} = \cot \frac{\pi}{\sqrt{x}} = \sqrt{x}$$

$$f(\sqrt{x}) = \sqrt{(\sqrt{x})^2 + 1} = \sqrt{x+1} = \frac{x}{x}$$

$$f \circ f\left(\frac{\sqrt{x}}{x}\right) = \boxed{2}$$

۱

$$\text{ا) } f \circ g\left(\frac{\pi}{x}\right) \rightarrow g\left(\frac{\pi}{x}\right) = x \cos^2 \frac{\pi}{x} = \frac{1}{x} \quad f\left(\frac{1}{x}\right) = f\left(\frac{1}{x}\right) \Rightarrow x=2 \rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{x-1}{x}} = \left(\frac{\sqrt{x}}{x}\right)$$

$$\text{ب) } f \circ g(\sqrt{x}) \rightarrow g(\sqrt{x}) = \frac{\sqrt{x}}{1-\sqrt{x}} = \frac{-\sqrt{x}(\sqrt{x}+1)}{x-1} = -x-\sqrt{x} \quad -\epsilon < -x-\sqrt{x} < -x$$

$$f(-x-\sqrt{x}) = [-x-\sqrt{x}] = \boxed{-\epsilon}$$

۲

$$g \circ f\left(\frac{\pi}{x}\right)$$

$$f\left(\frac{\pi}{x}\right) = \sin \frac{\pi}{x} = \frac{\sqrt{x}}{x}$$

$$g\left(\frac{\sqrt{x}}{x}\right) = \frac{\sqrt{x}}{x} \sqrt{1 - \left(\frac{\sqrt{x}}{x}\right)^2} = \frac{\sqrt{x}}{x} \sqrt{1 - \frac{x}{x^2}} = \frac{\sqrt{x}}{x} \sqrt{\frac{x-x}{x}} = \frac{\sqrt{x}}{x} \times \frac{1}{\sqrt{x}} = \boxed{\frac{1}{x}}$$

۳

$$\text{ا) } f \circ g = \{(x, \omega)(2, \omega)(4, 12)(8, 12)\} \quad x \rightarrow 4 \rightarrow \omega / 2 \rightarrow \epsilon \rightarrow \omega / 4 \rightarrow 8 \rightarrow 12 / 8 \rightarrow 10 \rightarrow 12$$

$$\text{ب) } g \circ f = \{\} = \emptyset \quad \epsilon \rightarrow \omega \rightarrow x / 4 \rightarrow \omega \rightarrow x / 8 \rightarrow 12 \rightarrow x / 10 \rightarrow 12 \rightarrow x$$

$$\text{ج) } f \circ f = \{\} = \emptyset \quad \epsilon \rightarrow \omega \rightarrow x / 4 \rightarrow \omega \rightarrow x / 8 \rightarrow 12 \rightarrow x / 10 \rightarrow 12 \rightarrow x$$

$$\text{د) } g \circ g = \{(x, 8)(2, 4)(4, 10)\} \quad \epsilon \rightarrow 4 \rightarrow 8 / 2 \rightarrow \epsilon \rightarrow 4 / 4 \rightarrow 8 \rightarrow 10 / 8 \rightarrow 10 \rightarrow x$$

۴

$$f = \{(2, 1)(3, 2)(\epsilon, \omega)(1, \nu)\} \quad g = \{(1, 2)(3, 1)(\alpha, \beta)(b, 1)\}$$

$$(x, 2) \in f \circ g \Rightarrow \boxed{x} \rightarrow 2 \rightarrow 2$$

$$a = \epsilon$$

$$b = \omega$$

$$\boxed{(a, b) = (\epsilon, \omega)}$$

$$(x, 1) \in g \circ f \Rightarrow x \rightarrow \boxed{x} \rightarrow 1$$

۵

$$f(f(n)) = a(an+b) + b = tnx+r \rightarrow a^2x + ab + b = tnx+r \rightarrow \begin{cases} a^2 = t \Rightarrow a = \pm r \\ ab + b = r \rightarrow b(a+1) = r \end{cases}$$

$$\Rightarrow a = r, b = 1$$

$$\Rightarrow a = -r, b = -1$$

$$\begin{aligned} f(n) &= rx + 1 \xrightarrow{x=-1} f(-1) = -r + 1 \\ f(n) &= -rx - r \xrightarrow{x=-1} f(-1) = -1 \end{aligned} \quad \left\{ \begin{array}{l} g(-1) \Rightarrow x = -r \\ g(\underbrace{r(-r)+r}_{-1}) = r(-r) - r = -1 \end{array} \right.$$

$$f(n) = \sqrt{x+|x|} \quad \begin{cases} x > 0 \rightarrow rx \\ x < 0 \rightarrow 0 \end{cases} \Rightarrow D_f = \mathbb{R} \quad g(n) = \frac{1}{x^2 - 2x} \Rightarrow D_g = \mathbb{R} - \{0, 2\}$$

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\} = \{x \in \mathbb{R} \mid \sqrt{x+|x|} \in \mathbb{R} - \{0, 2\}\}$$

$$\sqrt{x+|x|} \neq 0, 2 \rightarrow x \neq 1, x \neq 0 \Rightarrow D_{g \circ f} = (\cdot, +\infty) - \{1\}$$

$$D_f: [-1, 1] \quad D_g: (\cdot, +\infty) \Rightarrow D_{f \circ g} = D_f \cap D_g = [-1, 1]$$

$$D_{(f+g) \circ f} = \{x \in D_f \mid f(x) \in D_{f+g}\} = \{x \in [-1, 1] \mid 0 \leq \sqrt{1-x^2} \leq 1\}$$

$$0 \leq \sqrt{1-x^2} \leq 1 \rightarrow 1-x^2 \leq 1 \Rightarrow -x^2 \leq 0 \Rightarrow D_{(f+g) \circ f} = [-1, 1]$$

$$\text{وا) } f\left(\frac{rx+1}{x-r}\right) = tx+a \rightarrow \frac{rx+1}{x-r} = t \Rightarrow rx+1 = tx-rt \Rightarrow rx-tx = -rt-1 \rightarrow$$

$$x(r-t) = -rt-1 \rightarrow x = \frac{-rt-1}{r-t} \Rightarrow f(t) = \varepsilon\left(\frac{-rt-1}{r-t}\right) + a \rightarrow f(t) = \frac{-rt+1}{r-t}$$

$$\Rightarrow \boxed{f(n) = \frac{-rt+1}{r-t}}$$

$$\begin{aligned} \text{ب) } f\left(x + \frac{1}{x}\right) &= x^r + \frac{1}{x^r} \rightarrow \left(x + \frac{1}{x}\right)^r = x^r + rx^{r-1} \frac{1}{x} + rx \frac{1}{x^2} + \frac{1}{x^r} = x^r + rx + \frac{r}{x} + \frac{1}{x^r} \\ x^r + \frac{1}{x^r} &= \left(x + \frac{1}{x}\right)^r - r\left(x + \frac{1}{x}\right) \rightarrow f(t) = t^r - rt \Rightarrow \boxed{f(n) = x^r - rx} \end{aligned}$$

$$\begin{cases} g(r\sqrt{x}) = 0 \\ g(1) = 0 \end{cases}$$

$$g \circ f(n) = g(x\sqrt{x})$$

$g(n)$ به ازای x و $r\sqrt{x}$ مقادیر مشخصه است، $g \circ f(n)$ به ازای $x\sqrt{x}$ مقادیر مشخصه است.

$$f(n) = r\sqrt{x} \rightarrow x\sqrt{x} = r\sqrt{x} \Rightarrow x = r$$

$$f(n) = 1 \rightarrow x\sqrt{x} = 1 \Rightarrow x = 1$$

$$x_1 - x_r = r - 1 = \textcircled{1}$$