

$$f\left(\frac{\sqrt{x}}{x}\right) = \cot \frac{\pi \frac{\sqrt{x}}{x}}{\frac{x}{x}} = \cot \frac{\pi}{\sqrt{x}} = \sqrt{x}$$

$$f(\sqrt{x}) = \sqrt{(\sqrt{x})^2 + 1} = \sqrt{x+1} = \frac{x}{x}$$

$$f \circ f\left(\frac{\sqrt{x}}{x}\right) = \boxed{2}$$

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ا) $f \circ g\left(\frac{\pi}{x}\right) \rightarrow g\left(\frac{\pi}{x}\right) = x \cos^2 \frac{\pi}{x} = \frac{1}{x}$ $f\left(\frac{1}{x}\right) = f\left(\frac{1}{x}\right) \Rightarrow x=2 \rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{x-1}{x}} = \left(\frac{\sqrt{x}}{2}\right)$

ب) $f \circ g(\sqrt{x}) \rightarrow g(\sqrt{x}) = \frac{\sqrt{x}}{1-\sqrt{x}} = \frac{-\sqrt{x}(\sqrt{x}+1)}{x-1} = -2-\sqrt{x}$

$$-4 < -2-\sqrt{x} < -3$$

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$$f(-2-\sqrt{x}) = [-2-\sqrt{x}] = \boxed{-4}$$

$$g \circ f\left(\frac{\pi}{x}\right)$$

$$f\left(\frac{\pi}{x}\right) = \sin \frac{\pi}{x} = \frac{\sqrt{x}}{x}$$

$$g\left(\frac{\sqrt{x}}{x}\right) = \frac{\sqrt{x}}{x} \sqrt{1 - \left(\frac{\sqrt{x}}{x}\right)^2} = \frac{\sqrt{x}}{x} \sqrt{1 - \frac{x}{x^2}} = \frac{\sqrt{x}}{x} \sqrt{\frac{x-x}{x}} = \frac{\sqrt{x}}{x} \times \frac{1}{\sqrt{x}} = \boxed{\frac{1}{x}}$$

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ا) $f \circ g = \{(x, \infty), (2, \infty), (4, 12), (8, 12)\}$ $f \rightarrow 4 \rightarrow \infty / 2 \rightarrow \infty / 4 \rightarrow \infty / 8 \rightarrow 12$

ب) $g \circ f = \{\} = \emptyset$ $x \rightarrow \infty \rightarrow x / 4 \rightarrow \infty \rightarrow x / 8 \rightarrow 12 \rightarrow x$

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ج) $f \circ f = \{\} = \emptyset$ $x \rightarrow \infty \rightarrow x / 4 \rightarrow \infty \rightarrow x / 8 \rightarrow 12 \rightarrow x$

د) $g \circ g = \{(x, 8), (2, 4), (4, 10)\}$ $x \rightarrow 4 \rightarrow 8 / 2 \rightarrow 4 \rightarrow 10 / 4 \rightarrow 10 \rightarrow x$

$$f = \{(2, 1), (3, 2), (5, 5), (1, 7)\}$$

$$g = \{(1, 2), (3, 1), (5, 3), (6, 1)\}$$

$$(5, 2) \in f \circ g \Rightarrow \boxed{f} \rightarrow 2 \rightarrow 2$$

$$a = 2$$

$$b = 5$$

$$(a, b) = (2, 5)$$

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$$(5, 1) \in g \circ f \Rightarrow 1 \rightarrow \boxed{g} \rightarrow 1$$

$$f(f(n)) = a(an+b) + b = t_n + r \rightarrow a^2x + ab + b = t_n + r \rightarrow \begin{cases} a^2 = 2 \Rightarrow a = \pm\sqrt{2} \\ ab + b = r \rightarrow b(a+1) = r \end{cases}$$

$$\Rightarrow a = \sqrt{2}, b = 1$$

$$\Rightarrow a = -\sqrt{2}, b = -1$$

$$f(n) = \sqrt{2}n + 1 \xrightarrow{n=-1} f(-1) = 0$$

$$f(n) = -\sqrt{2}n - 1 \xrightarrow{n=-1} f(-1) = 0$$

$$\left\{ \begin{array}{l} g(-1) \Rightarrow x = -\sqrt{2} \\ g(\sqrt{2}(-\sqrt{2}) + 1) = \sqrt{2}(-\sqrt{2}) - 1 = -2 \end{array} \right.$$

$$f(n) = \sqrt{x+|x|} \begin{cases} x > 0 \rightarrow \sqrt{x} \\ x \leq 0 \rightarrow 0 \end{cases} \Rightarrow D_f = \mathbb{R} \quad g(n) = \frac{1}{x^2 - 2x} \Rightarrow D_g = \mathbb{R} - \{0, 2\}$$

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\} = \{x \in \mathbb{R} \mid \sqrt{x+|x|} \in \mathbb{R} - \{0, 2\}\}$$

$$\sqrt{x+|x|} \neq 0, 2 \rightarrow x \neq 1, x \neq 0 \Rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$$

$$D_f = [-1, 1] \quad D_g = (0, +\infty) \Rightarrow D_{f \circ g} = D_f \cap D_g = [-1, 1]$$

$$D_{(f+g) \circ f} = \{x \in D_f \mid f(x) \in D_{f+g}\} = \{x \in [-1, 1] \mid 0 \leq \sqrt{1-x^2} \leq 1\}$$

$$0 \leq \sqrt{1-x^2} \leq 1 \rightarrow 1-x^2 \leq 1 \Rightarrow -x^2 \leq 0 \Rightarrow D_{(f+g) \circ f} = [-1, 1]$$

$$\text{وا) } f\left(\frac{rx+1}{x-r}\right) = tx + a \rightarrow \frac{rx+1}{x-r} = t \Rightarrow rx+1 = tx - rt \Rightarrow rx - tx = -rt - 1 \rightarrow$$

$$x(r-t) = -rt - 1 \rightarrow x = \frac{-rt-1}{r-t} \Rightarrow f(t) = \frac{-rt-1}{r-t} + a \rightarrow f(t) = \frac{-rt+11}{r-t}$$

$$\Rightarrow f(n) = \frac{-11n+11}{r-n}$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + 2x \cdot \frac{1}{x} + \frac{1}{x^2} = x^2 + 2 + \frac{1}{x^2} = x^2 + \frac{1}{x^2} + 2 \rightarrow f(t) = t^2 - 2 \Rightarrow f(n) = x^2 - 2x$$

$$\begin{cases} g(\sqrt{2}) = 0 \\ g(1) = 0 \end{cases}$$

$$g \circ f(n) = g(x\sqrt{x})$$

$g(n)$ به ازای n و $x\sqrt{x}$ مقادیر میگیرد، $g \circ f(n)$ به ازای $f(n)$ و $x\sqrt{x}$ مقادیر میگیرد.

$$f(n) = x\sqrt{x} \rightarrow x\sqrt{x} = \sqrt{2} \Rightarrow x = 2$$

$$f(n) = 1 \rightarrow x\sqrt{x} = 1 \Rightarrow x = 1$$

$$x_1 - x_2 = 2 - 1 = 1$$