

Subject:

Year: Month: Day: ()

کتابت: تاریخ: ۹/۰۳/۱۴۰۰

page: ()

$$f(n) = \begin{cases} \cot \frac{\pi n}{4} & ; n \leq 1 \\ \sqrt{n^2 + 1} & ; n > 1 \end{cases} \Rightarrow f(f(1)) \xrightarrow{\frac{1}{4} < 1} \cot \frac{\pi \cdot 1}{4} \Rightarrow \cot \frac{\pi}{4} = 1 \quad \leftarrow ①$$

$$f(\sqrt{3}) = \sqrt{3^2 + 1} = 2$$

②

(الف)

$$f(g(\frac{x}{y})) \Rightarrow g(\frac{x}{y}), \quad 2 \cos^2 \frac{\pi x}{y} = \frac{1}{y} \Rightarrow f(\frac{1}{y}) = \sqrt{\frac{y^2 - 1}{y}} = \frac{\sqrt{y}}{y}$$

$$f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{\sqrt{2} + 2}{1 - 2} = -2 - \sqrt{2} \Rightarrow f(-2 - \sqrt{2}) = [-2 - \sqrt{2}] \quad \leftarrow ③$$

10

$$\Rightarrow f(-2 - \sqrt{2}) = -3$$

$$g(f(\frac{x}{y})) \Rightarrow f(\frac{x}{y}) = \sin \frac{\pi x}{y} = \frac{\sqrt{2}}{y} \Rightarrow g(\frac{\sqrt{2}}{y}) = \frac{\sqrt{2}}{y} \sqrt{1 - \frac{1}{y}} \Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{y}$$

$$\frac{y}{x} = \frac{1}{y}$$

④

$$\text{الف) } f(g(m)) \rightarrow x \rightarrow y \rightarrow d \rightarrow (x, d), \quad y \rightarrow x \rightarrow d \rightarrow (y, d), \quad y \rightarrow 1 \rightarrow 12 \rightarrow (y, 12) \\ 1 \rightarrow 10 \rightarrow 12 \rightarrow 11, 12 \Rightarrow f(g(m)) = \{(x, d), (y, d), (y, 12), (11, 12)\}$$

$$\text{ب) } g(f(m)) \rightarrow \{ \} \quad \text{و) } g(g(m)) = \{(x, 1), (y, 9), (y, 10)\}$$

20

$$\text{ج) } f(f(m)) \rightarrow \{ \}$$

$$(x, y) \in f(g(m)) \rightarrow x \rightarrow y \rightarrow z \Rightarrow a = x \quad \text{و) } (a, b) = (x, d) \quad \text{و) } (x, 1) \in g(f(m)) \rightarrow x \rightarrow d \rightarrow 1 \Rightarrow b = d$$

25

$$f(n) = an + b \Rightarrow f(an + b) = a(an + b) + b = a^2n + ab + b \quad \text{و) } f(n) = a^2n + ab + b$$

$$\Rightarrow f(n) = a^2n + ab + b$$

$$a^2 = 3 \Rightarrow a = \pm \sqrt{3} \Rightarrow -b + b = 3 \Rightarrow b = -3 \quad \text{و) } 3b + b = 3 \Rightarrow b = 1$$

$$g(m) = a'm + b' \Rightarrow g(xm + y) = a'(xm + y) + b' \Rightarrow a'xm + a'y + b' = xm + y$$

$$\Rightarrow ya = 3 \Rightarrow a = \frac{3}{y}, \quad x \times \frac{3}{y} + b' = -y \Rightarrow b' = -y - \frac{3}{y} = -\frac{y^2 + 3}{y}$$

$$\Rightarrow g(n) = \frac{3}{y}n - \frac{y^2 + 3}{y}, \quad f(n) = 3n + 1 \quad \text{و) } f(n) = -2n - 3$$

PAYCO

$$\Rightarrow g \circ f(-1) = [-1]$$

$$-\frac{14}{y} = -1$$

Subject:

Year: Month: Day: ()

page: ()

$$D_f = n + |n| \geq 0 \Rightarrow \begin{matrix} n < 0 \\ n > 0 \end{matrix} \Rightarrow \begin{matrix} n < 0 \\ n > 0 \end{matrix} \Rightarrow n \neq 0$$

⑤

$$\frac{n \neq 0}{n^2 - 1} \Rightarrow n^2 - 1 \neq 0 \Rightarrow n \neq 0, n \neq \pm 1 \Rightarrow f(n) \notin D_g \Rightarrow$$

$$\sqrt{n+|n|} \neq 0, \sqrt{n+|n|} \neq 1 \Rightarrow \sqrt{n} \neq 1 \Rightarrow n \neq 1 \rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$$

$\hookrightarrow n \neq 0$

$$D_f(n) \leq 1 - n^2 \geq 0 \Rightarrow n^2 \leq 1 \Rightarrow n \leq 1, n \geq -1 \Rightarrow [-1, 1] \quad \text{①}$$

$$f(f(n)) + g(f(n)) \rightarrow f(f(n)) \rightarrow [-1, 1] \text{ دالة } f, \quad g(f(n)) \rightarrow [0, 1] \in [-1, 1]$$

$$10 \Rightarrow D_{f \circ g} \subset [-1, 1]$$

$$\text{ii) } f(n+1) = f(n) + \alpha \Rightarrow \frac{n+1}{n-1} = t \Rightarrow n+1 = t(n-1) \Rightarrow n+1 = tn - t \Rightarrow n(1-t) = -t-1 \Rightarrow n = \frac{-t-1}{1-t} \Rightarrow$$

$$f\left(\frac{-t-1}{1-t}\right) = f\left(\frac{-t-1}{1-t}\right) + \alpha \Rightarrow f(n) = \frac{n-1}{n-1}, \quad (n \neq 1) \quad t \neq 1$$

②

$$15 \Rightarrow f(t) = t^3 - 1 \Rightarrow f(n) = n^3 - 1$$

$$\left(n + \frac{1}{n}\right)^3 = n^3 + \frac{1}{n^3} - 3\left(n + \frac{1}{n}\right)$$

$$g(f(n)) \Rightarrow g(n\sqrt{n}) \Rightarrow n=1 \rightarrow g(1\sqrt{1}),$$

③

$$n=1 \Rightarrow g(1) \Rightarrow 1-1=0$$