

(20)

کتابت: دفتر کتب

9-2-3

page: ()

Subject:

Year: Month: Day: ()

$$f(n) = \begin{cases} \cot \frac{\pi n}{4} & ; n \leq 1 \\ \sqrt{n^2+1} & ; n > 1 \end{cases} \Rightarrow f(f(1)) \xrightarrow{\frac{1}{4} < 1} \cot \frac{\pi \cdot 1}{4} \Rightarrow \cot \frac{\pi}{4} = 1 \quad \text{①}$$

$$f(\sqrt{3}) = \sqrt{3^2+1} = 2 \quad \text{②}$$

الف) 5

$$f(g(\frac{x}{y})) \Rightarrow g(\frac{x}{y}), \quad 2 \cos^2 \frac{\pi x}{y} = \frac{1}{y} \Rightarrow f(\frac{1}{y}) = \sqrt{\frac{y^2-1}{y}} = \frac{\sqrt{y}}{y} \quad \text{③}$$

$$f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -2-\sqrt{2} \Rightarrow f(-2-\sqrt{2}) = [-2-\sqrt{2}] \quad \text{④}$$

10

$$\Rightarrow f(-2-\sqrt{2}) = -2$$

$$g(f(\frac{x}{y})) \Rightarrow f(\frac{x}{y}) = \sin \frac{\pi x}{y} = \frac{\sqrt{2}}{y} \Rightarrow g(\frac{\sqrt{2}}{y}) = \frac{\sqrt{2}}{y} \sqrt{1 - \frac{1}{y^2}} \Rightarrow \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{y} = \frac{\sqrt{2}}{y} \quad \text{⑤}$$

$$\frac{y}{x} = \frac{1}{y}$$

الف) 15

$$\text{الف) } f(g(n)) \rightarrow x \rightarrow y \rightarrow a \rightarrow (x, a), \quad y \rightarrow x \rightarrow a \rightarrow (y, a), \quad y \rightarrow 1 \rightarrow 12 \rightarrow (y, 12) \\ 1 \rightarrow 10 \rightarrow 12 \rightarrow 11, 12 \Rightarrow f(g(n)) = \{(x, a), (y, a), (y, 12), (11, 12)\}$$

$$\text{ب) } g(f(n)) \rightarrow \{ \} \quad \text{و) } g(g(n)) = \{(x, 1), (y, 9), (y, 10)\}$$

20

$$\text{ج) } f(f(n)) \rightarrow \{ \}$$

$$(x, y) \in f(g(n)) \rightarrow x \rightarrow y \rightarrow z \Rightarrow a = x \quad \text{و) } (a, b) = (x, a) \quad \text{⑥}$$

$$(x, 1) \in g(f(n)) \rightarrow x \rightarrow a \rightarrow 1 \Rightarrow b = a$$

25

$$f(n) = an + b \Rightarrow f(an + b) = a(an + b) + b = a^2n + ab + b \quad \text{⑦}$$

$$\Rightarrow fn + 1 = a^2n + ab + b \Rightarrow$$

$$a^2 = 1 \Rightarrow a = \pm 1 \Rightarrow -1b + b = 1 \Rightarrow b = -1 \quad \text{و) } 1b + b = 1 \Rightarrow b = \frac{1}{2}$$

$$g(n) = a'n + b' \Rightarrow g(xn + 1) = a'(xn + 1) + b' = a'xn + a' + b' = xn + 1$$

$$\Rightarrow 1a' = 1 \Rightarrow a' = 1, \quad 1 \times \frac{1}{2} + b' = -1 \Rightarrow b' = -1 - \frac{1}{2} = -\frac{3}{2}$$

$$\Rightarrow g(n) = \frac{1}{2}n - \frac{3}{2}, \quad f(n) = 2n + 1 \quad \text{و) } f(n) = -2n - 1$$

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$$\Rightarrow g \circ f(-1) = -1$$

$$-\frac{14}{2} = -7$$

Subject:

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page: ()

$$Df = n + |n| \geq 0 \Rightarrow \begin{matrix} n < 0 \\ n > 0 \end{matrix} \Rightarrow \begin{matrix} n < 0 \\ n > 0 \end{matrix} \Rightarrow n \neq 0$$

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$$\frac{n > 0}{n^2 - r} \Rightarrow n^2 - r \neq 0 \Rightarrow n \neq 0, n \neq r \Rightarrow f(n) \notin Dg \Rightarrow$$

$$\sqrt{n+|n|} \neq 0, \sqrt{n+|n|} \neq r \Rightarrow \sqrt{r} \neq r \Rightarrow r \neq 1 \rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$$

$\hookrightarrow n \neq 0$

$$Df(n) \leq 1 - n^2 \geq 0 \Rightarrow n^2 \leq 1 \Rightarrow n \leq 1, n \geq -1 \Rightarrow [-1, 1]$$

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$$f(f(n)) + g(f(n)) \rightarrow f(f(n)) \rightarrow [-1, 1] \text{ (مجموعة)}, g(f(n)) \rightarrow [0, 1] \in [-1, 1]$$

$$10 \Rightarrow D_{f+g} \subset [-1, 1]$$

$$11) f(rn+1) = rn + \alpha \Rightarrow \frac{rn+1}{n-r} = t \Rightarrow rn+1 = t(n-r) \Rightarrow$$

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$$rn - tn = -rt - 1 \Rightarrow n(r-t) = -rt - 1 \Rightarrow n = \frac{-rt-1}{r-t} \Rightarrow n = \frac{-rt+1}{t-r}$$

$$f\left(\frac{-rt+1}{t-r}\right) = f\left(\frac{-rt+1}{t-r}\right) + \alpha \Rightarrow f(n) = \frac{rn-1}{n-r}, (n \neq r)$$

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$$15) f(t) = t^r - rt \Rightarrow f(n) = n^r - rn$$

$$\left(n + \frac{1}{n}\right)^r = n^r + \frac{1}{n^r} - r\left(n + \frac{1}{n}\right)$$

$$g(f(n)) \Rightarrow g(n\sqrt{n}) \Rightarrow n=r \rightarrow g(r\sqrt{r}),$$

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$$n=1 \Rightarrow g(1) \Rightarrow r-1=1$$