تمرین ۱

۱-

$$f(x)=\begin{cases}\cot\frac{\pi x}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$$

$$\Rightarrow \cot\frac{\frac{\pi}{4}}{\frac{1}{2}}\to \cot\frac{\pi}{2}=\sqrt{2}$$

$$fof \longrightarrow f(\sqrt{2})=\sqrt{2+1}=2$$

۲-

الف) $f(\frac{1}{x})=\sqrt{\frac{1-x^2}{x^2}}$ $\quad g(x)=2\cos^2 x$

$fog(\frac{\pi}{4})=?$

$\hookrightarrow g(\frac{\pi}{4})=2\cos^2\frac{\pi}{4}=\frac{1}{2}\to f(\frac{1}{\frac{1}{2}})=F(2)=\frac{\sqrt{3}}{2}$

ب) $f(x)=[x]$ $\quad g(x)=\frac{x}{x-1}$

$fog(\sqrt{2})=?$

$\hookrightarrow g(\sqrt{2})=\frac{\sqrt{2}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}=\frac{\sqrt{2}+2}{2-1}=-\sqrt{2}-2$

$f(-\sqrt{2}-2)=[-\sqrt{2}-2]=[-3,-4]\Rightarrow -4$

۳-

$f(x)=\sin x$ $\quad g(x)=x\sqrt{1-x^2}$

$gof(\frac{\pi}{3})=?$

$\hookrightarrow f(\frac{\pi}{3})=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}\to g(\frac{\sqrt{3}}{2})=\frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

۴-

الف) $fog\to\{(12,7),(11,6),(10,5),(9,4)\}$

ب) $gof\to$ تعریف نشده

ج) $fof\to$ تعریف نشده

د) $gog\to\{(10,6),(6,5),(7,4)\}$

۵- $f=\{(2,1),(3,2),(4,3),(1,4)\}$

$g=\{(1,2),(2,1),(a,4),(b,1)\}$

$f\circ g=(4,2)$ $\quad g\circ f=(4,1)$

$(a,b)=?$

$(a,b)=(4,3)$

به صورت معکوس عمل می‌کنیم

$a=4$ ← $b=3$ ←

۶- $f(f(x))=4x+3$ $\quad g(2x+1)=2x-1$ $\quad g\circ f(-1)=?$

$\hookrightarrow f(f(x))=f(ax+b)\rightarrow a^2=4\rightarrow a=\pm 2$

$ab+b=3\longrightarrow b(a+1)=3$

$a=-2\rightarrow b=-3$ ← $\quad$ $\hookrightarrow a=2\rightarrow b=1$

$2x+1=y\longrightarrow g(y)=y-2+\frac{y-1}{2}=\frac{3y-3}{2}$

$\hookrightarrow f(x)=2x+1$ یا $-2x-3$

$f(-1)=-1$ $\quad f(-1)=-1\Rightarrow g(-1)=-3=g\circ f(-1)$

۷- $f(x)=\sqrt{x+|x|}$ $\quad g(x)=\frac{1}{x^2-4x}$

D $g\circ f=?$

f تعریف $\rightarrow x+|x|\geq 0\rightarrow x\geq 0$

g تعریف $\rightarrow x^2-4x\neq 0\longrightarrow x\neq 0$ یا $x\neq 4\Rightarrow x\neq\{0,4\}$

$D_{g\circ f}\Rightarrow(0,4)\cup(4,+\infty)$

۸- $f(x)=\sqrt{1-x^2}$ $\quad g(x)=\sqrt{x}$ $\quad D(f+g)of$

f دامنه $\to 1-x^2 \geq 0 \to -1 \leq x \leq 1$ } دامنه $(f+g) = [0,1]$

g دامنه $\to x \geq 0$

$0 \leq f(x) \leq 1 \to \sqrt{1-x^2} \leq 1 \to x^2 \geq 0$ همواره

دامنه $(f+g)of \to [-1,1]$

۹- الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5$

$\downarrow$ $x \neq 2$ $\to y=\frac{3x+1}{x-2} \to yx-2y=3x+1 \to x=\frac{2y+1}{y-3}$

$f(t) = \frac{4(2y+1)}{y-3} + \frac{5(y-3)}{y-3} = \frac{13y-11}{y-3} = f(x) = \frac{13x-11}{x-3}$

ب) $f(x+\frac{1}{x}) = x^3+\frac{1}{x^3}$

توان ۳ $= x^3 + \frac{3x^2}{x} + \frac{3x}{x^2} + \frac{1}{x^3}$

$\to x^3+\frac{1}{x^3} = (x+\frac{1}{x})^3 - 3(x+\frac{1}{x})$

[illegible] $y=(x+\frac{1}{x})$ را در نظر بگیریم (طبق مطالب جزوه):

$f(y)= y^3-3y \to f(x) = x^3-3x$

۱۰- $x=1, 2\sqrt{2}$ $\quad f(x)=x\sqrt{x}$

$gof(x)=?$

$\hookrightarrow g(x\sqrt{x}) \to g(2\sqrt{2})=0 \to x=2$

$\hookrightarrow g(1)=0 \to x=1$

اختلاف $=2-1=1$