

$$F(x) = \begin{cases} \cot \frac{\pi x}{4} & x \leq 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases} \rightarrow F\left(\frac{1}{4}\right) = \cot\left(\frac{\frac{1}{4} \pi}{1}\right) = \cot\left(\frac{\pi}{4}\right) \rightarrow \cot\left(\frac{\pi}{4}\right)$$

$$\rightarrow F(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = \sqrt{2+1} = \sqrt{3} = \frac{\cos(\frac{\pi}{4})}{\sin(\frac{\pi}{4})} = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = \sqrt{2}$$

الف) $F\left(\frac{1}{x}\right) = \sqrt{\frac{2x-1}{x^2}}$ و $g(x) = 2 \cos^2 x \rightarrow f \circ g\left(\frac{\pi}{4}\right) \rightarrow g\left(\frac{\pi}{4}\right) = 2 \cos^2\left(\frac{\pi}{4}\right)$

$$\hookrightarrow F\left(\frac{1}{f}\right) = \sqrt{\frac{2(2) - 1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

ب) $f(x) = [x]$ و $g(x) = \frac{x}{1-x} \rightarrow \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{1-2} = \frac{\sqrt{2}+2}{-1} = -(\sqrt{2}+2) = -2-\sqrt{2}$

$$f[-(\sqrt{2}+2)] = (-4)$$

$F(x) = \sin x$ و $g(x) = x \sqrt{1-x^2} \rightarrow g \circ f\left(\frac{\pi}{4}\right) \rightarrow F = \sin \frac{\pi}{4} \rightarrow \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$$\rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} \rightarrow \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$$

الف) $(f \circ g)(x) = \{(4, 5), (2, 5), (4, 12), (1, 12)\}$

ب) $g \circ f(x) = \emptyset$

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د) $g \circ g(x) = \{(4, 1), (2, 4), (4, 10)\}$

$f = \{(2, 1), (3, 2), (4, 5), (1, 7)\}$ و $g = \{(1, 2), (2, 1), (4, 3), (5, 1)\}$

$(4, 2) \in f \circ g \rightarrow f(g(4)) = 2$ ، $a = 4 \rightarrow g(4) = 3 \rightarrow a = 4$ و $(4, 1) \in g \circ f \rightarrow g(5) = 1 \rightarrow (5, 1) \rightarrow b = 5$

$\rightarrow (a, b) = (4, 5)$

$$f(x) = \sqrt{x+1} \Rightarrow g(x) = \frac{1}{x^2 - 1} \neq 0$$

$$\text{وإذن } \rightarrow x \geq 0 \Rightarrow \sqrt{x+1} = \sqrt{x} \rightarrow \text{وإذن } \rightarrow x < 0 \Rightarrow \sqrt{x+1} = 0 \rightarrow \text{وإذن } \rightarrow \mathbb{R}$$

$$\rightarrow g \circ f = \rightarrow \frac{1}{(x+1) - \sqrt{x+1}} \neq 0 \Rightarrow \text{وإذن } \rightarrow \mathbb{R}^+ \setminus \{1\}$$

$$\Rightarrow D_f = \mathbb{R}^+ \setminus \{1\}$$

$$f(f(x)) = f(x+1) \rightarrow f(x+1) = \sqrt{x+1} + 1 \rightarrow f(-1) = \sqrt{-1+1} + 1 = 1$$

$$g(2x+1) = 2x-1 \rightarrow a(ax+b)+b = fa+1 \rightarrow a^2x+ab+b = 2x+1$$

$$g \circ f(-1) = \frac{1}{-1-1} = -\frac{1}{2} \quad a^2 = 2 \rightarrow a = \pm \sqrt{2} \rightarrow 2x+1 = 2x+1 \rightarrow b = -1$$

$$f(x) = 2x+1 = 2(-1)+1 = -1 \rightarrow f(-1) = -1$$

$$f(a) = -2a-1 = -2(-1)-1 = 1 \rightarrow f(a) = 1$$

$$\rightarrow 2a+1 = -1 \rightarrow 2a = -2 \rightarrow a = -1$$

$$f(x) = \sqrt{1-x^2} \Rightarrow g(x) = \sqrt{x}$$

$$1-x^2 \geq 0 \rightarrow -1 \leq x \leq 1$$

$$(f+g) \circ f \rightarrow (f+g)(f(x)) = f(f(x)) + g(f(x)) \Rightarrow g(\sqrt{1-x^2}) = \sqrt{\sqrt{1-x^2}} \rightarrow \sqrt{1-x^2} \geq 0$$

$$f(\sqrt{1-x^2}) = \sqrt{1-(\sqrt{1-x^2})^2} \rightarrow \sqrt{x^2} = |x|$$

$$\text{وإذن } \rightarrow [-1, 1]$$

$$\text{أ) } f\left(\frac{2x+1}{x-1}\right) = 2x+1 \rightarrow t = \frac{2x+1}{x-1} \rightarrow f(t) = 2x+1$$

$$t(x-1) = 2x+1 \rightarrow tx - t = 2x+1 \rightarrow tx - 2x = t+1 \rightarrow x(t-2) = t+1 \rightarrow x = \frac{t+1}{t-2}$$

$$\rightarrow x = \frac{t+1}{t-2} \rightarrow t = 2 \rightarrow \text{وإذن } \rightarrow f(t) = f\left(\frac{t+1}{t-2}\right) = 2 \rightarrow \frac{1 \cdot t + 1}{t-2} = 2 \rightarrow f(x) = \frac{1 \cdot t - 11}{t-2} = \frac{1 \cdot x - 11}{x-2}$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \rightarrow t = x + \frac{1}{x} \rightarrow f(t) = x^2 + \frac{1}{x^2} \rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \rightarrow x^2 + \frac{1}{x^2} = t^2 - 2 \rightarrow f(x) = x^2 - 2x$$

$$\rightarrow x^2 = x^2 + \frac{1}{x^2} + 2x = x^2 + \frac{1}{x^2} + 2x \rightarrow f(x) = x^2 - 2x$$

$$g(1) = 0 \Rightarrow g(\sqrt{x}) = 0 \Rightarrow f(x) = x\sqrt{x} \rightarrow x > 0$$

$$g \circ f(x) = 0 \rightarrow f(x) = 1 \rightarrow x = 1 \rightarrow f(x) = x\sqrt{x} \rightarrow x = 1 \rightarrow 1 - 1 = 0$$