

$$f(n) = \begin{cases} \cot \frac{\pi n}{4} & ; n < 1 \\ \sqrt{n^2 + 1} & ; n \geq 1 \end{cases} \Rightarrow f \circ f\left(\frac{1}{4}\right) = ? \Rightarrow f\left(f\left(\frac{1}{4}\right)\right) = f\left(\frac{1}{4}\right) = \cot \frac{\pi}{4} = \sqrt{3}$$

$$f(\sqrt{3}) = \sqrt{3+1} = 2 \Rightarrow f \circ f\left(\frac{1}{4}\right) = 2$$

$$\text{الف) } g(n) = 2 \cos^2 n, f\left(\frac{1}{n}\right) = \sqrt{\frac{2n-1}{n^2}} \Rightarrow f \circ g\left(\frac{\pi}{4}\right)$$

$$g\left(\frac{\pi}{4}\right) = 2 \cos^2 \frac{\pi}{4} = 2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{2}, f\left(\frac{1}{\frac{1}{2}}\right) = \sqrt{\frac{2-1}{\frac{1}{4}}} = \frac{\sqrt{2}}{\frac{1}{2}} = 2$$

$$\text{ب) } f(n) = [n], g(n) = \frac{n}{1-n}, f \circ g(\sqrt{2}) = ?, g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} \Rightarrow \frac{\sqrt{2}+2}{1-2} \Rightarrow -\sqrt{2}-2, f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -4$$

$$f(n) = \sin n, g(n) = n\sqrt{1-n^2}, g \circ f\left(\frac{\pi}{4}\right) = ? \Rightarrow f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{2}{4}} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$$

$$g(n) = \{(4, 4), (2, 4), (4, 1), (1, 1)\}, f(n) = \{(4, 6), (4, 6), (1, 12), (1, 12)\}$$

$$\text{الف) } f \circ g(n) = \{(4, 6), (2, 6), (4, 12), (1, 12)\}$$

$$\text{ب) } g \circ f(n) = \emptyset$$

$$\text{ج) } f \circ f = \emptyset$$

$$\text{د) } g \circ g(n) = \{(4, 1), (2, 4), (4, 1)\}$$

$$f = \{(1, 1), (2, 2), (4, 6), (1, 1)\}, g = \{(1, 2), (2, 1), (a, 2), (b, 1)\}$$

$$(1, 2) \in f \circ g, (4, 1) \in g \circ f, (a, b) = ? \Rightarrow (1, 2) \in f \circ g \Rightarrow f(g(1)) = 2 \Rightarrow f(2) = 2$$

$$g(1) = 2 \Rightarrow a = 2$$

$$(4, 1) \in g \circ f \Rightarrow g(f(4)) = 1 \Rightarrow g(6) = 1 \Rightarrow f(6) = 6 \Rightarrow b = 6$$

$$(a, b) = (2, 6)$$

$$f(f(n)) = 2n+3 \Rightarrow 2n+3 = a(an+b)+b \Rightarrow 2n+3 = a^2n + ab + b \Rightarrow a = 2$$

$$2b+b = 3 \Rightarrow b = 1, g \circ f(-1) = g(f(-1)) = 2n+1 \Rightarrow -2+1 = -1$$

$$g(-1) = 2n+3 = -1 \Rightarrow n = -2 \Rightarrow g(-1) = 2 \times (-2) + 3 = -1$$

$$f(n) = \sqrt{n+|n|}, g(n) = \frac{1}{n^2 - 4n}, D_{g \circ f} = ? \quad (9)$$

$$n+|n| \geq 0 \Rightarrow |n| \geq -n \Rightarrow D_f = \mathbb{R} = \text{مباين صحيح}$$

$$D_g: n^2 - 4n \neq 0 \Rightarrow n(n-4) \neq 0 \Rightarrow n \neq \begin{matrix} 4 \\ 0 \end{matrix} \Rightarrow D_g = \mathbb{R} - \{0, 4\}$$

$$D_{g \circ f} = \{n \in D_f \mid f(n) \in D_g\} \Rightarrow D_{g \circ f} = \mathbb{R} - \{0, 4\}$$

$$f(n) = \sqrt{1-n^2}, g(n) = \sqrt{n} \Rightarrow D_{(f+g) \circ f} = ? \quad (10)$$

$$D_f: 1-n^2 \geq 0 \Rightarrow 1 \geq n^2 \Rightarrow -1 \leq n \leq 1 \Rightarrow D_f = [-1, 1]$$

$$D_g: n \geq 0 \Rightarrow D_g = [0, +\infty)$$

$$D_{f+g} \Rightarrow D_f \cap D_g \Rightarrow [0, 1] \quad D_{(f+g) \circ f} = \{n \in D_f \mid f(n) \in D_{f+g}\} \Rightarrow$$

$$\{n \in [-1, 1] \mid f(n) \in [0, 1]\} \Rightarrow \{n \in [-1, 1] \mid \underbrace{0 \leq \sqrt{1-n^2} \leq 1}_{\text{مباين برقرار}}\} \Rightarrow$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$\text{مثال 1) } f\left(\frac{r+1}{n-2}\right) = f(n) + 6 \Rightarrow f(t) = f\left(\frac{r+1}{t-2}\right) + 6 \Rightarrow f(t) = \frac{13t-11}{t-2} \quad (9)$$

$$\frac{r+1}{n-2} = t \Rightarrow n = \frac{r+1}{t-2}$$

$$f(n) = \frac{13n-11}{n-2}$$

$$\text{ب) } f\left(n + \frac{1}{n}\right) = n^3 + \frac{1}{n^3} \Rightarrow f\left(n + \frac{1}{n}\right) = \left(n + \frac{1}{n}\right)\left(n^3 - 1 + \frac{1}{n^3}\right) \Rightarrow$$

$$\left(n + \frac{1}{n}\right)\left(n + \frac{1}{n}\right)^3 \Rightarrow f(n) = n^3 - 3n$$

$$g(\sqrt{2}) = 0, g(1) = 0, f(n) = n\sqrt{n}, g \circ f(n) = 0 \quad (10)$$

$$\left. \begin{array}{l} g(f(n)) = 0 \rightarrow f(n) = 1 \\ \rightarrow f(n) = \sqrt{2} \end{array} \right\} \Rightarrow \left. \begin{array}{l} 1 = n\sqrt{n} \Rightarrow n = 1 \\ \sqrt{2} = n\sqrt{n} \Rightarrow n = 2 \end{array} \right\} \Rightarrow 1, 2$$