19,75

نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس ..... A

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$f(x)\begin{cases}\cot\frac{\pi x}{4} & ;x\le 1 \quad (1)\\ \sqrt{x^2+1} & ;x>1 \quad (2)\end{cases}$

$f(\frac{2}{3}) = \cot\frac{\pi\times\frac{2}{3}}{4} = \cot\frac{\pi}{6} = \sqrt{3}$

$f(\sqrt{2}) = \sqrt{(\sqrt{2})^2+1} = \sqrt{2+1} = \sqrt{3} = (2)$

$\frac{2}{3} \to \cot\frac{\pi}{6} \to \sqrt{3} \to (2)$ (ب)

$\frac{2}{3} < 1$ پس در رابطه (1) جایگذاری می شود.

$\sqrt{2} > 1$ پس در رابطه (2) جایگذاری می شود.

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الف) $g(\frac{\pi}{3}) = 2\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{2}$; $f(\frac{1}{2}) = \sqrt{\frac{2\times2-1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$ (ج)

$\frac{\pi}{3} \to \frac{1}{2} \to \frac{\sqrt{3}}{2}$

ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}(1+\sqrt{2})}{-1} \Rightarrow -\sqrt{2}-2 \Rightarrow -(\sqrt{2}+2)$

$f(-(\sqrt{2}+2)) = [-\sqrt{2}-2] \Rightarrow (-4)$

$\sqrt{2} \to -\sqrt{2}-2 \to -4$

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$f(\frac{\pi}{3}) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow g(x) = x\sqrt{1-x^2} \Rightarrow g(x) = \frac{\sqrt{3}}{2}\sqrt{1-(\frac{\sqrt{3}}{2})^2} \to$

$\frac{\sqrt{3}}{2}\sqrt{1-\frac{3}{4}} \Rightarrow \frac{\sqrt{3}}{2}\sqrt{\frac{1}{4}} \Rightarrow \frac{\sqrt{3}}{2}\times\frac{1}{2} = \frac{\sqrt{3}}{4} = (\frac{1}{2})$ (ج)

$\frac{\pi}{3} \to \frac{\sqrt{3}}{2} \to \frac{1}{2}$

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الف) $fog(x) \Rightarrow \begin{cases}4\to\not{8}\to a \Rightarrow (4,a)\\ 2\to 4\to a \Rightarrow (2,a)\\ 6\to 8\to 12 \Rightarrow (6,12)\\ 8\to 10\to 12 \Rightarrow (8,12)\end{cases}$ (ج)

$fof(x)\begin{cases}4\to a\to\emptyset\\ 6\to a\to\emptyset\\ 8\to 12\to\emptyset\\ 10\to 12\to\emptyset\end{cases}$ $f.f(x)=\emptyset$ ✓

ب) $gof(x) \Rightarrow \begin{cases}4\to a\to 8\\ 6\to a\to 8\\ 8\to 12\to 8\\ 10\to 12\to 8\end{cases}$ $gof(x)=\emptyset$

$gog(x)\begin{cases}4\to 6\to 8 \Rightarrow (4,8)\\ 2\to 4\to 6 \Rightarrow (2,6)\\ 6\to 8\to 10 \Rightarrow (6,10)\\ 8\to 10\to\emptyset\end{cases} = gog(x)$ (ج)

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$fog(x)$; $(a,b) = (4,a)$

$1\to 2\to 1$ $(1,1)$

$4\to 1\to 7$ $(4,7)$

$a\to 4\to 2$ $(a,2) \Rightarrow (4,2)$

$b\to 1\to 7$ $(b,7) \Rightarrow (1,7)$

$\boxed{a=6}$ $\boxed{b=a}$

$gof(x)$

$2\to 1\to 2$

$4\to 2\to$

$6\to a\to 1$ و $\boxed{b=a}$

$1\to 7\to$

(ج)

$f(f(x)) = 4x+3 \Rightarrow f(ax+b) = a(ax+b)+b \Rightarrow a^2x+ab+b \Rightarrow$ $4x+3 \Rightarrow \boxed{a=2}\ \boxed{b=1}$ $g(2x+3) = 3x-2 \Rightarrow a(2x+3)+b = 3x-2 \Rightarrow 2ax+3a+b=$ $3x-2 \Rightarrow a=1/5 \quad b=-6/5 \quad y=1/5x-6/5$ $g(f(-1)) \Rightarrow g(-1) \Rightarrow \boxed{-8}$ (۲)	۶
$f(x) = \sqrt{x+\|x\|} \qquad g(x) = \frac{1}{x^2-4x} \Rightarrow x(x-4) \Rightarrow x \neq 0, 4$ (۲) $gof = \{x \in D_f\ ;\ f(x) \in D_g\} \quad \sqrt{x+\|x\|} \neq 0 \Rightarrow x+\|x\| \geq 0 \Rightarrow$ $\boxed{x \leq 0}\ ,\ \sqrt{x+\|x\|} \neq 4 \rightarrow \boxed{x=8}\quad D_{gof} = \mathbb{R} - [(-\infty,0] \cup \{8\}]$	۷
$f(x) = \sqrt{1-x^2} \qquad g(x) = \sqrt{x}$ $D_g = [0,+\infty) \Rightarrow D_{f+g} = D_{f \cap g} = [0,1]$ (۱٫۷۵) $D_{(f+g)of} = \{x \in D_f\ ,\ f(x) \in D_{g+f}\} \Rightarrow \{x \in [-1,1]$ و $0 \leq \sqrt{1-x^2} \leq 1 \Rightarrow 1-x^2 \leq 1 \Rightarrow -x^2 \leq 0 \Rightarrow x^2 \geq 0$ (همیشه برقرار) $\mathbb{R}$ $D_{(f+g)of} = [0,1]$ $[-1,1]$	۸
الف) $\frac{3x+1}{x-2} = t \Rightarrow 3x+1 = tx-2t \Rightarrow x(3-t) = -2t-1 \Rightarrow$ $\boxed{x = \frac{2t+1}{t-3}} \Rightarrow f(t) = 4\left(\frac{2t+1}{t-3}\right)+5 \Rightarrow f(x) = \frac{8x+4}{x-3}+5$ $\left(\frac{13x-11}{x-3}\right)$ (۲) ب) $x+\frac{1}{x} = t \Rightarrow (t)^3 = x^3+\frac{1}{x^3}+3x+\frac{3}{x} \Rightarrow \left(x+\frac{1}{x}\right)^3 - \frac{3}{x} - 3x \Rightarrow$ $-3\left(\frac{1}{x}+x\right)$ $t^3-3t \Rightarrow \boxed{f(x) = x^3-3x}$	۹
$gof(x) \rightarrow g(f(x)) \rightarrow g(x\sqrt{x}) = 0$ $x\sqrt{x} = 2\sqrt{2} \Rightarrow (x=2) \Rightarrow x\sqrt{x} = 1 \Rightarrow (x=1)$ (۲) $\Delta x = 2-1 = ①$	۱۰