ماتیاها و بیان ۲۰ تکلیف ۹۰ر

۱. $F(F(\frac{3}{4}))$ $\cot\frac{\pi \frac{3}{4}}{4} = \cot\frac{1}{6}\pi \Rightarrow \sqrt{3}$

$F(\sqrt{3})$ $\sqrt{(\sqrt{3})^2+1} = \boxed{2}$

(۲)

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۲. الف) $F(g(\frac{\pi}{3})) \Rightarrow g(\frac{\pi}{3}) = 2(\frac{1}{2})^2 = \frac{1}{2}$

$F(\frac{1}{2}) = \sqrt{\frac{2(2)-1}{4}} = \boxed{\frac{\sqrt{3}}{2}}$

(۲)

ب) $F(g(\sqrt{2})) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = \frac{\sqrt{2}+2}{-1} = -2-\sqrt{2} \Rightarrow F(-2-\sqrt{2}) = \boxed{-4}$

$[-2-\sqrt{2}]$

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۳) $g(F(\frac{\pi}{4})) \Rightarrow F(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-(\frac{\sqrt{2}}{2})^2} = \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2}\sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2} = \frac{2}{4} = \boxed{\frac{1}{2}}$

(۲)

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۴) الف) $4 \xrightarrow{g(x)} 5 \xrightarrow{g(x)} 5$

$2 \to 4 \to 5$

$6 \to 8 \to 12$

$8 \to 10 \to 12$

$\Rightarrow Fog(x) = \{(4,5),(2,5),(6,12),(8,12)\}$

(۲)

ج) $4 \xrightarrow{F(x)} 5\times$

$6 \to 5\times$

$8 \to 12\times$

$10 \to 12\times$

$Fof(x) = \varnothing$

ب) $4 \xrightarrow{F(x)} 5$

$6 \to 5$

$8 \to 12$

$10 \to 12$

$g \circ F(x) = \varnothing$

د) $4 \xrightarrow{g(x)} 6 \xrightarrow{g(x)} 8$

$2 \to 4 \to 6$

$6 \to 8 \to 10$

$8 \to 10$

$g \circ g(x) = \{(4,8),(2,6),(6,10)\}$

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۵) $2 \xrightarrow{F(x)} 1 \to 2$

$3 \to 2$

$4 \to a \to 1 \to (b,1)$ $b=5$

$1 \to 7$

gof ب

$1 \to 2 \to 1$

$3 \to 1 \to 7$

$a \to 3 \to 2 \to$ $(4,2) = (a,2)$ $4=a$

$b \to 1 \to 7$

$(a,b) = \boxed{(4,5)}$

(۲)

$a(ax+b)+b=4x+3$

$a^2x+ab+b=4x+3 \Rightarrow a=2 \;,\; b=1 \Rightarrow g=2x+1$

$c(2x+3)+b=3x-2 \Rightarrow 2cx+3c+b=3x-2 \Rightarrow c=\frac{3}{2} \quad b=-\frac{13}{2} \Rightarrow g'=\frac{3}{2}x-\frac{13}{2}$

$g(f(-1)) \Rightarrow 2(-1)+1=-1 \Rightarrow g(-1)=\frac{3}{2}(-1)-\frac{13}{2}=-\frac{3}{2}-\frac{13}{2}=-\frac{16}{2}=\boxed{-8}$

۷.

$x\in D_f : f(x)\in D_g \qquad D_f=\sqrt{x+|x|} \Rightarrow D_f=\mathbb{R}^+$

$\boxed{D_{g\circ f}=(0,8)\cup(8,+\infty)}$

$\hookrightarrow \sqrt{x+|x|}\neq 4 \hookrightarrow x\neq 8$

$\sqrt{x+|x|}\neq 0 \quad x\neq 0$

$\frac{1}{x^2-4x} \rightarrow \neq 0 \Rightarrow x\neq 0 \;,\; x\neq 4$

۸.

$x\in D_f : f(x)\in D_{f+g} \Rightarrow D_f: -1\leq x\leq 1$

$\hookrightarrow 1-x^2\geq 0$

$f(x)\in D_{f+g} \Rightarrow [0,1] \hookrightarrow 0\leq f(x)\leq 1$

$\hookrightarrow 0\leq 1-x^2\leq 1 \qquad -1\leq x\leq 1$

$D_f=[-1,1] \;\cap\; D_g=[0,+\infty)$

$\boxed{[-1,1]}$

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۹. (الف)

$\frac{3x+1}{x-2}=t \Rightarrow xt-2t=3x+1 \Rightarrow xt-3x=2t+1$

$x(t-3)=2t+1 \Rightarrow x=\frac{2t+1}{t-3}$

$f(t)=4\left(\frac{2t+1}{t-3}\right)+5=\frac{8t+4}{t-3}+\frac{5t-15}{t-3}=\frac{13t-11}{t-3} \Rightarrow f(x)=\boxed{\frac{13x-11}{x-3}}$

ب.

$x+\frac{1}{x}=t \qquad t^3=x^3+3x^2\frac{1}{x}+3x\frac{1}{x^2}+\frac{1}{x^3}$

$\Rightarrow t^3=x^3+3x+\frac{3}{x}+\frac{1}{x^3} \Rightarrow t^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

$\Rightarrow f(x)=t^3-3t \Rightarrow f(x)=\boxed{x^3-3x}$

۱۰.

$g(1)=0 \qquad g(2\sqrt{2})=0$

$g(f(x))=0$

$\Rightarrow f(x)=1 \Rightarrow x\sqrt{x}=1 \Rightarrow x_1=1$

$f(x_2)=2\sqrt{2} \Rightarrow x\sqrt{x}=2\sqrt{2} \quad x_2=2$

$|x_1-x_2|=|2-1|=\boxed{1}$